THE EFFECTS OF ATMOSPHERIC REFRACTION ON THE ACCURACY OF LASER RANGING SYSTEMS

by

D. L. Zanter
C. S. Gardner
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RRL Publication No. 471

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Supported By

Contract No. NASA NSG 5049

NATIONAL AERONAUTICS & SPACE ADMINISTRATION
Goddard Space Flight Center
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(NASA-CR-146313) THE EFFECTS OF ATMOSPHERIC REFRACTION ON THE ACCURACY OF LASER RANGING SYSTEMS (Illinois Univ.) 151 p HC $6.75 CSCL 20E

N76-18435

Unclas
G3/36 18439

UILU-ENG-76-2538

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1. INTRODUCTION

Laser ranging systems have developed rapidly in the past few years. Limitations in ranging accuracy are caused by atmospheric refraction and scattering. In this paper the effects of atmospheric refraction will be discussed.

Atmospheric refraction will increase the optical path length to an orbiting satellite by a few meters when the satellite is near zenith, and by over ten meters when the satellite is near 10° elevation. Saastamoinen [1] and Marini [2] have developed correction formulas to compensate for the effects of atmospheric refraction on the optical path length. These formulas are convenient for correcting satellite tracking data since they require only surface measurements of pressure, temperature, and relative humidity. Marini and Murray [3] investigated the accuracy of their formula by comparing the results to ray traces through refractivity profiles derived from radiosonde balloon measurements of pressure, temperature, and relative humidity. The difference between the range corrections obtained by ray tracing and the correction formula was only a few millimeters at 10° elevation, and decreased to less than one millimeter near zenith. The correction formulas derived by Saastamoinen and Marini, and the ray traces through the refractivity profiles all assume a spherically symmetric refractivity profile. The errors introduced by this assumption are investigated in this paper by ray tracing through three-dimensional profiles. The results of this investigation indicate that the difference between ray traces through the spherically symmetric and three-dimensional profiles is approximately three centimeters at 10° and decreases to less than one half of a centimeter at 80°. Therefore, if the accuracy desired in future laser ranging systems is less than a few centimeters, Saastamoinen and Marini's formulas must be altered to account for the fact that the refractivity profile is not spherically symmetric.

2. MARINI AND MURRAY'S FORMULA

Marini and Murray's formula is based upon the theory of geometric optics. The optical path length is defined as the integral of the group index of refraction along the ray path

$$R_e = \int_c n_g \, d\ell \quad . \tag{2-1}$$

The ray path will lie entirely in a plane if the refractivity is spherically symmetric. Using this assumption, it follows that $d\ell = dr/\sin\theta$. This fact simplifies equation (2-1) to

$$R_e = \int_{r_0}^{r_1} \frac{n_g}{\sin\theta} dr \tag{2-2}$$

where r_1, r_0, and θ are shown in Figure 1. The equation for group refractivity [3] can be written as

$$N_g = 80.343 \; f(\lambda) \frac{P}{T} - 11.3 \frac{e}{T} \tag{2-3}$$

where

$$f(\lambda) = .9650 + \frac{.0164}{\lambda^2} + \frac{.000228}{\lambda^4} \tag{2-4}$$

$$e = \frac{Rh}{100} \times 6.11 \times 10^{\frac{7.5(T - 273.15)}{237.3 + (T - 273.15)}} \tag{2-5}$$

and

λ = wavelength of radiation (microns)

P = atmospheric pressure (mb)

T = temperature (^{o}K)

Rh = relative humidity (%).

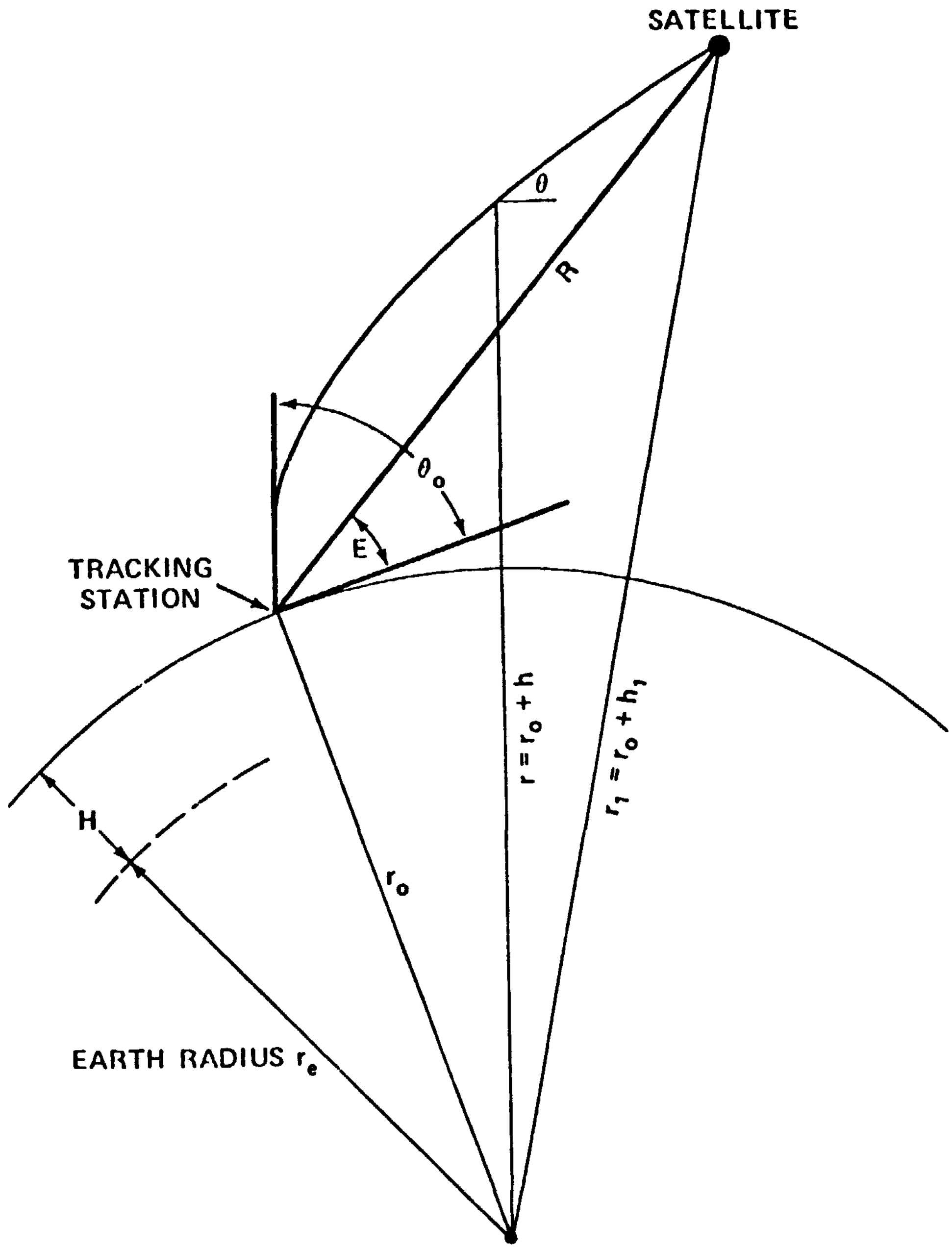

Figure 1. Geometry of laser ranging site and satellite target [3].

The group index of refraction is given by

$$n_g = 1 + 10^{-6} N_g \quad . \tag{2-6}$$

The desired range correction is the difference between the measured path length, R_e, and the straight path length, R.

$$\Delta R = R_e - R \tag{2-7}$$

Using equations (2-2), (2-4), and (2-7) the range correction can be represented as the following integral:

$$\Delta R = 10^{-6} \int_{r_0}^{r_1} \frac{N_g}{\sin\theta}\, dr + \left[\int_{r_0}^{r_1} \frac{dr}{\sin\theta} - R \right] \quad . \tag{2-8}$$

The first term can be considered the velocity error. The bracketed term is the difference between the geometric lengths of the ray and straight line paths. To evaluate these integrals Marini and Murray [3] used the perfect gas law, the law of partial pressures, the hydrostatic equations, and Snell's law for a spherically stratified medium. They obtained the following formula for ΔR

$$\Delta R = \frac{f(\lambda)}{f(\phi,H)} \, \frac{A + B}{\sin E + \dfrac{B/(A + B)}{\sin E + .01}} \tag{2-9}$$

where

$$f(\phi,H) = 1 - .0026 \cos 2\phi - .00031H \tag{2-10}$$

$$K = 1.163 - .00968 \cos 2\phi - .00104T_0 + .00001435P_0 \tag{2-11}$$

$$A = .002357P_0 + .000141e_0 \tag{2-12}$$

$$B = (1.084 \times 10^{-8})\, P_0 T_0 K + (4.734 \times 10^{-8}) \frac{P_0^2}{T_0} \frac{2}{(3 - 1/K)} \tag{2-13}$$

and

P_0 = surface pressure at site (mb)

T_0 = surface temperature at site (°K)

ϕ = latitude of site

H = elevation of site above sea level (km) .

Notice that the formula only requires surface measurements. The integrand of equation (2-8) was expanded in inverse powers of $\sin\theta_0$ to obtain equation (2-9). The constant .01 in formula (2-9) was derived empirically to compensate for the neglect of higher order terms in this expansion.

Marini and Murray investigated the accuracy of their formula by comparing the results to those obtained by ray tracing through spherically symmetric refractivity profiles. The profiles were obtained by using radiosonde balloon data to calculate the group refractivity using equation (2-3). The refractivity was only a function of height since one radiosonde balloon was used in its calculation. The results of their investigation are summarized in Table 1. RT_1 is the correction obtained by ray tracing through the spherically symmetric refractivity profiles while MM is the correction obtained from Marini and Murray's formula [equation (2-9)].

TABLE 1

RESULTS OBTAINED BY MARINI AND MURRAY FOR RUBY LASER (λ = 694 nanometers)

Elevation Angle	RT_1 Mean (m)	RT_1 - MM	
		Mean (cm)	Standard Deviation (cm)
80°	2.47	.07	.04
40°	3.69	-.1	.07
20°	6.91	-.05	.12
10°	13.32	-.08	.49

3. NONSYMMETRICAL REFRACTIVITY PROFILE

In our investigation a three-dimensional refractivity profile was generated from radiosonde balloons which were released simultaneously from three separate locations around the ranging site. Radiosonde data from Project Haven Hop 1 [4] was used. Figure 2 gives the locations of the balloon release sites, which were centered around Washington, D.C. After the balloons were released, they were frequently blown down range as far as 150 kilometers. An overhead view of three typical balloon trajectories is pictured in Figure 3. Notice that the three trajectories are very similar. This indicates that at least the wind conditions at the three sites are relatively homogeneous.

Three tables of pressure, temperature, and relative humidity versus height were generated from the radiosonde data. Each table corresponded to a different balloon release. An example of the data from one radiosonde is shown in Figure 4. The data was taken from a height equivalent to the site's altitude to approximately 15 kilometers. Each table's measurements were normalized to standard heights to simplify the ray tracing program (see Appendix A). This was done by interpolating the pressure, temperature, relative humidity, azimuth, and elevation between two data points of the balloon release. The temperature, relative humidity, azimuth, and elevation measurements were linearly interpolated. The azimuth and elevation at a particular height represent the balloon's position with respect to the tracking station from which the balloon was released. Using the hydrostatic equation, an interpolating formula for pressure can be obtained[3]

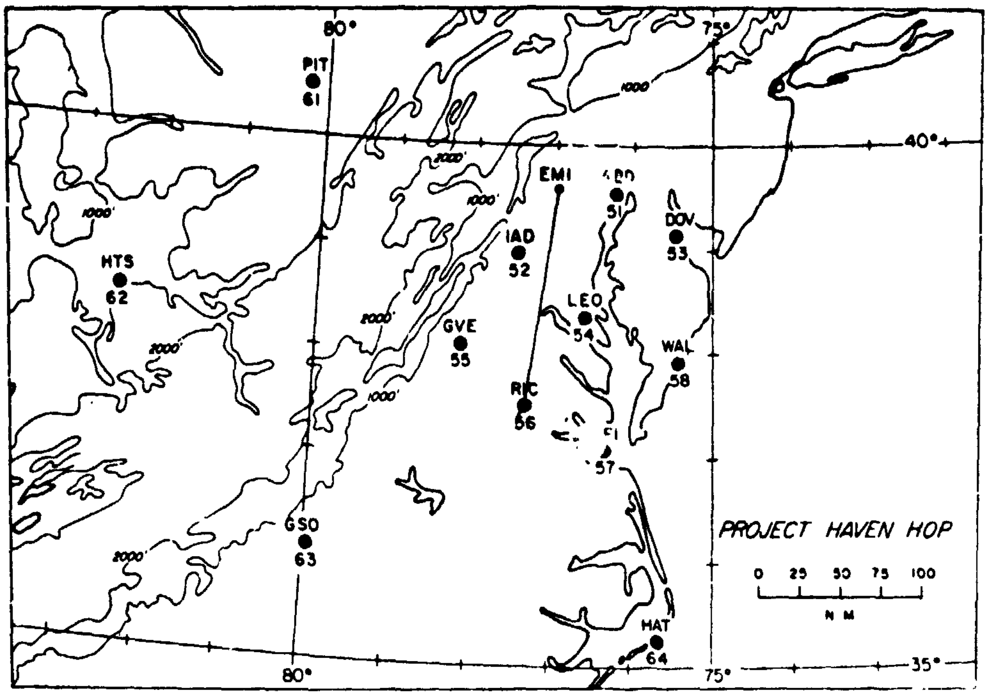

Figure 2. Locations of balloon release sites for Project Haven Hop 1 [4].

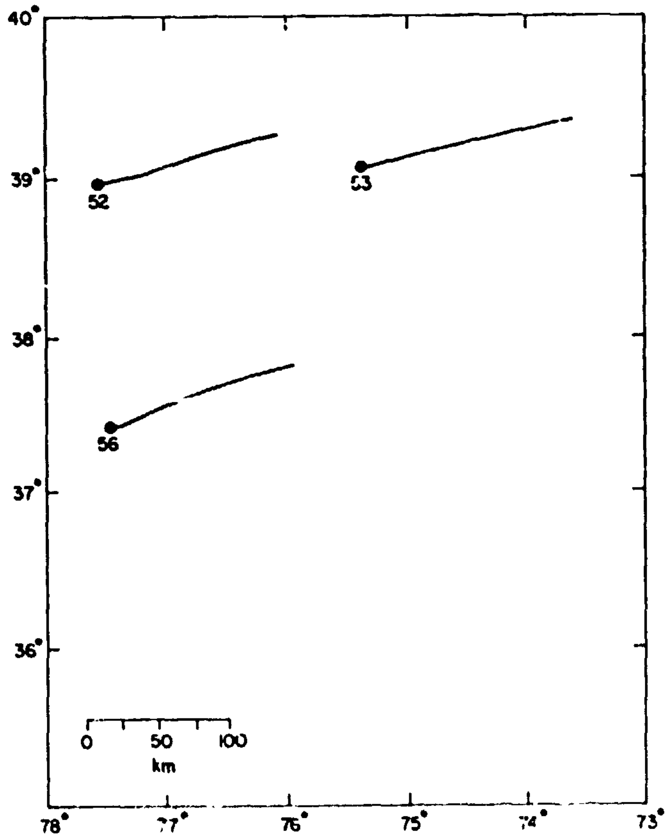

Figure 3. Balloon trajectories for sites 52, 53, and 56. Release time = 17.30. Release ·te = 2/13/70.

WAVELENGTH IN MICRONS = .530000E+00

STATION NO =55 LATITUDE= 38.071 DEGREES LONGITUDE = -78.136 DEGREES

TEMP	RELHUM	PRESS	HEIGHT	AZIMUTH	ELEVATION
.27084E+03	.60237E+02	.99700E+03	.14020E+03	.21502E+00	.91316E+00
.27752E+03	.60946E+02	.98700E+03	.23740E+03	.22569E+03	.76630E+02
.27647E+03	.59012E+02	.97200E+03	.36186E+03	.18349E+03	.71660E+02
.27572E+03	.58341E+02	.95500E+03	.50479E+03	.15257E+03	.66310E+02
.27453E+03	.59793E+02	.94000E+03	.63257E+03	.15186E+03	.65400E+02
.27355E+03	.60606E+02	.92600E+03	.75321E+03	.15395E+03	.63980E+02
.27243E+03	.60711E+02	.91100E+03	.88396E+03	.15963E+03	.63880E+02
.27158E+03	.62041E+02	.89800E+03	.99866E+03	.16138E+03	.61610E+02
.27022E+03	.69387E+02	.88500E+03	.11145E+04	.16185E+03	.59880E+02
.26945E+03	.71715E+02	.87200E+03	.12317E+04	.16169E+03	.58400E+02
.26859E+03	.74662E+02	.86000E+03	.13410E+04	.16057E+03	.55930E+02
.26859E+03	.71716E+02	.84500E+03	.14517E+04	.15919E+03	.52720E+02
.27349E+03	.58455E+02	.83000E+03	.15644E+04	.15930E+03	.48720E+02
.27073E+03	.33293E+02	.82000E+03	.17177E+04	.15944E+03	.46520E+02
.27011E+03	.30417E+02	.80600E+03	.18542E+04	.15915E+03	.45590E+02
.26929E+03	.29218E+02	.79100E+03	.20027E+04	.15978E+03	.45260E+02
.26872E+03	.30902E+02	.77800E+03	.21334E+04	.15915E+03	.44530E+02
.26735E+03	.33901E+02	.76400E+03	.22760E+04	.15815E+03	.43360E+02
.26648E+03	.44558E+02	.74800E+03	.24416E+04	.15487E+03	.41980E+02
.26463E+03	.51499E+02	.73300E+03	.25993E+04	.15121E+03	.40510E+02
.26352E+03	.56743E+02	.71900E+03	.27487E+04	.14773E+03	.37980E+02
.26287E+03	.59686E+02	.70800E+03	.28676E+04	.14508E+03	.35600E+02
.26169E+03	.56433E+02	.69800E+03	.29770E+04	.14246E+03	.33230E+02
.26107E+03	.51221E+02	.69000E+03	.30655E+04	.13988E+03	.30800E+02
.26130E+03	.45456E+02	.68200E+03	.31550E+04	.13752E+03	.28840E+02
.26102E+03	.35976E+02	.67200E+03	.32680E+04	.13561E+03	.26890E+02
.26108E+03	.23161E+02	.66400E+03	.33596E+04	.13372E+03	.25330E+02
.26165E+03	.15021E+02	.65500E+03	.34639E+04	.13211E+03	.23600E+02
.25990E+03	.17686E+02	.64500E+03	.35813E+04	.13037E+03	.22410E+02
.25892E+03	.18693E+02	.63600E+03	.36881E+04	.12927E+03	.21220E+02
.25796E+03	.19462E+02	.62600E+03	.38081E+04	.12832E+03	.20270E+02
.25713E+03	.18593E+02	.61500E+03	.39418E+04	.12723E+03	.19620E+02
.25612E+03	[illegible]	.60600E+03	.40526E+04	.12597E+03	.18730E+02

Figure 4. Sample of radiosonde data. Release site = 55. Release time = 17:30.
Release date = 1/20/70.

$$\frac{P_2}{P_1} = \left(\frac{T_{V1}}{T_{V2}}\right)^{\frac{GM(H_2 - H_1)}{R(T_{V2} - T_{V1})}} \tag{3-1}$$

where

$G = 9.8$ m/sec^2

M = molecular weight of dry air = 28.966

R = universal gas constant = 8314.36 joules $(°K)^{-1}$ $(\text{kg-mole})^{-1}$

H_2 = height of desired pressure (km)

H_1 = height of known pressure (km)

T_{V1} = virtual temperature at H_1 (°K)

T_{V2} = virtual temperature at H_2 (°K)

P_1 = known pressure (mb)

P_2 = desired pressure (mb) .

The virtual temperature is related to T (°K) [3] by

$$T_V = \frac{T}{1 - 0.379 \frac{e}{P}} \tag{3-2}$$

where e is given as before by formula (2-6) and P is the pressure in mb. Assuming that virtual temperature varies linearly with respect to height, the value of T_{V2} can be linearly interpolated between two data points. Therefore the desired pressure value P_2 can be found by using (3-1). Using these interpolation formulas, tables of pressure, temperature, and relative humidity can be generated to a height as high as the final radiosonde data point. Above this height the pressure is assumed to be a decaying exponential, while the remaining measurements are assumed to be the value of the last measured data point. This is the same approach that Marini and Murray used to construct their pressure, temperature, and relative humidity tables from radiosonde

data [3]. The difference between Marini and Murray and our investigation is that three tables are constructed. From these tables, the refractivity at any point along the path of each balloon trajectory can be calculated using (2-3). If the refractivity is desired at a height not equal to a standard height, the values of pressure, temperature, and relative humidity are interpolated. The interpolating is done between the two standard heights closest to the desired height. These interpolations are identical to the interpolations used in creating the tables. Using these interpolated values the refractivity can be calculated by applying formulas (2-3), (2-4), and (2-5).

To generate a 3-dimensional refractivity profile, we will assume the refractivity varies linearly in the horizontal direction. The refractivity at any point can be obtained by interpolating between the refractivities calculated at the same height along the trajectory of each of the three balloons using the formula

$$N(\underline{r}) = N_r + \theta \cdot N_\theta + \phi \cdot \sin(\theta) \cdot N_\phi \tag{3-3}$$

where θ and ϕ are the co-latitude and longitude of the vector $\underline{r} = (r,\theta,\phi)$. The geometry of the problem is illustrated in Figure 5. Notice in Figure 6 that $\theta \cdot r$ is proportional to horizontal displacement in the north-south direction while $\phi \cdot \sin(\theta) \cdot r$ is proportional to horizontal displacement in the east-west direction. Since a linear variation of group refractivity in the horizontal east-west and north-south directions was assumed, equation (3-3) follows.

The coefficients N_r, N_θ, and N_ϕ at any height, h, are calculated by solving the matrix equation

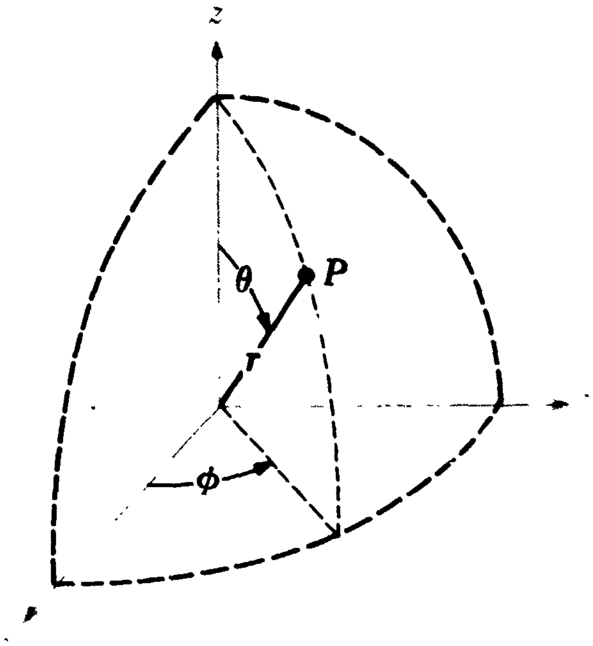

Figure 5. Spherical coordinate system.

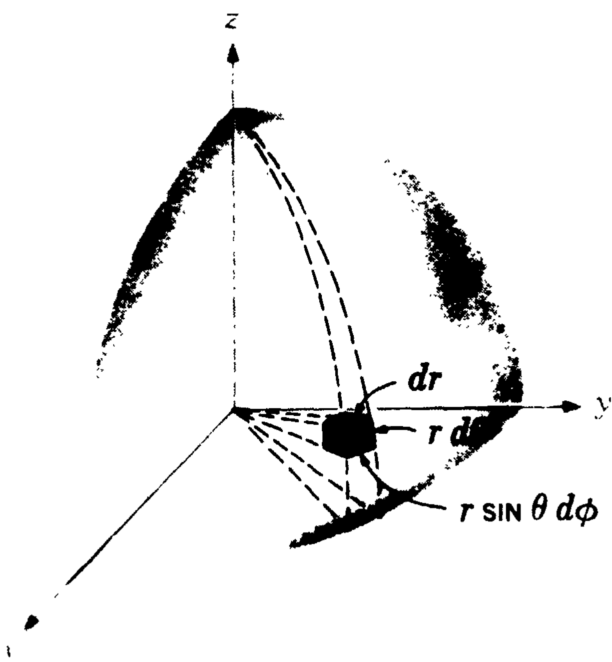

Figure 6. Differential elements in spherical coordinates.

$$\begin{bmatrix} N_r \\ N_\theta \\ N_\phi \end{bmatrix} = \begin{bmatrix} 1 & \theta_1 & \phi_1 \sin \theta_1 \\ 1 & \theta_2 & \phi_2 \sin \theta_2 \\ 1 & \theta_3 & \phi_3 \sin \theta_3 \end{bmatrix}^{-1} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix} \tag{3-4}$$

where N_1, N_2, and N_3 are the refractivities measured by the three radiosonde balloons at the given altitude, h, and (θ_1,ϕ_1), (θ_2,ϕ_2), and (θ_3,ϕ_3) are the co-latitude and longitude of the balloons at the given altitude. r is related to h by

$$r = h + r_0 \tag{3-5}$$

where r_0 is the radius of the earth at the laser site (see Figure 1). N_r, N_θ, and N_ϕ vary as a function of r but are constant with respect to θ and ϕ.

The gradients with respect to r, θ, and ϕ can be calculated by differentiating (3-3)

$$\frac{\partial N}{\partial r} = \frac{\partial N_r}{\partial r} + \theta \cdot \frac{\partial N_\theta}{\partial r} + \phi \cdot \sin(\theta) \cdot \frac{\partial N_\phi}{\partial r} \tag{3-6}$$

$$\frac{\partial N}{\partial \theta} = N_\theta + \phi \cdot \cos(\theta) \cdot N_\phi \tag{3-7}$$

$$\frac{\partial N}{\partial \phi} = \sin(\theta) \cdot N_\phi \tag{3-8}$$

The refractivity at any position (h,θ,ϕ) can now be calculated. The values of pressure, temperature, relative humidity, azimuth, and elevation angle along the balloon trajectories are obtained from the three tables at a height h. These values are calculated by interpolating between two standard heights.

The azimuth and elevation angle are converted to longitude and co-latitude ($\phi_1, \phi_2, \phi_3, \theta_1, \theta_2, \theta_3$). Equations (2-3) through (2-5) are used along with the interpolated values of pressure, temperature, and relative humidity to obtain the group refractivities at a height h along each balloon trajectory. The coefficients N_r, N_θ, and N_ϕ are calculated using equation (3-4) at a height h. These coefficients are then used in equation (3-3) along with θ and ϕ to obtain $N(h, \theta, \phi)$. By this method the three-dimensional refractivity profile is created.

4. ANALYSIS OF THREE-DIMENSIONAL REFRACTIVITY PROFILE AND GRADIENTS

The nonsymmetrical aspects of group refractivity are discussed in this section. If group refractivity was spherically symmetric, the refractivities measured along each balloon trajectory should be equal at equivalent heights. Figures 7 and 8 are plots of the difference between the group refractivities measured along each balloon trajectory versus height. Figures 7 and 8 are on pages 15 and 16. Figure 7 used release sites 52, 55, and 58, while Figure 8 used sites 53, 55, and 57. Notice Figure 8 has a larger difference between the refractivities than Figure 7.

The refractivities at heights directly above each balloon release station were calculated using equation (3-3). Figure 9 illustrates the geometry of the problem.

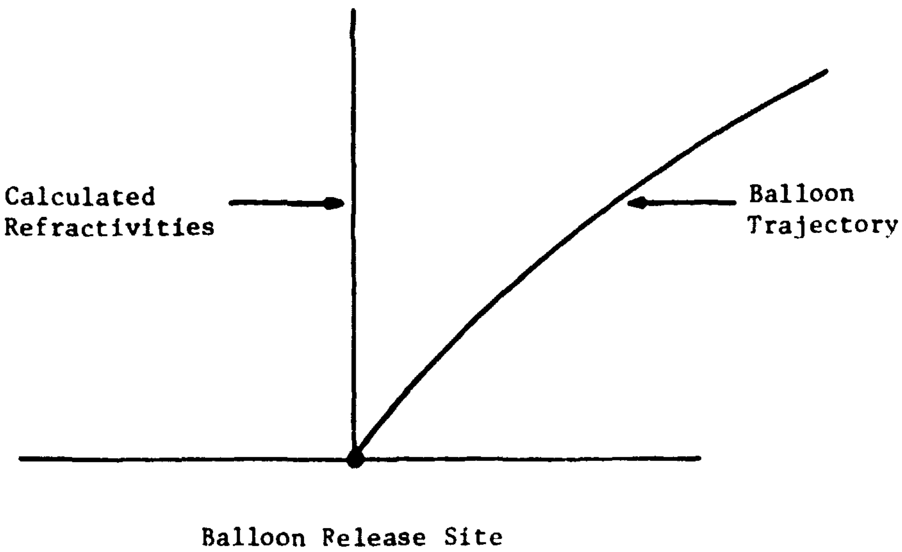

Figure 9. Geometry of refractivity calculations directly above each balloon release site.

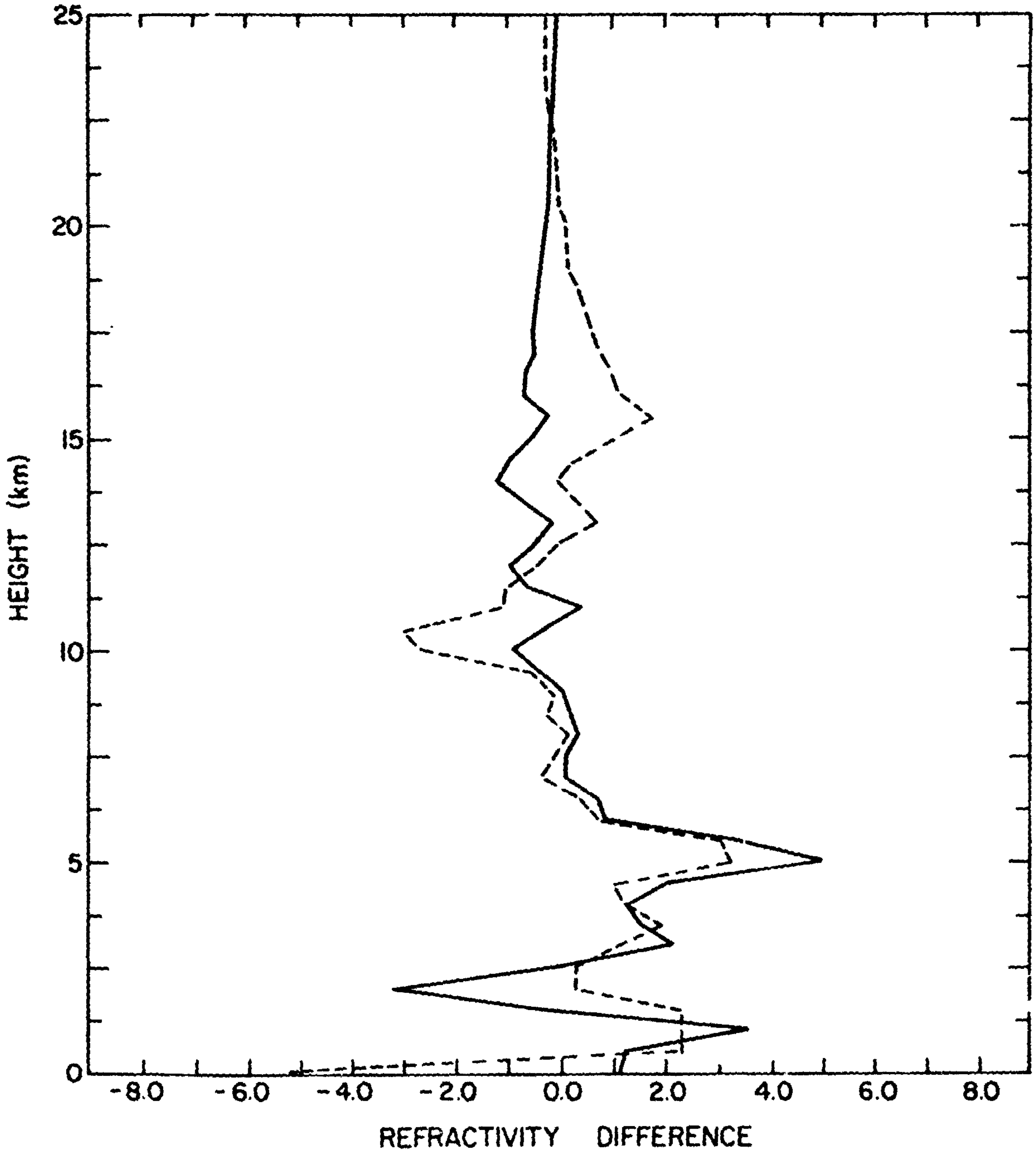

Figure 7. Difference between group refractivities measured along balloon trajectories versus height. Site 1 = 52. Site 2 = 55. Site 3 = 58. Time = 6:30. Date = 1/27/70. —— $(N_{g1} - N_{g2})$, - - - $(N_{g1} - N_{g3})$.

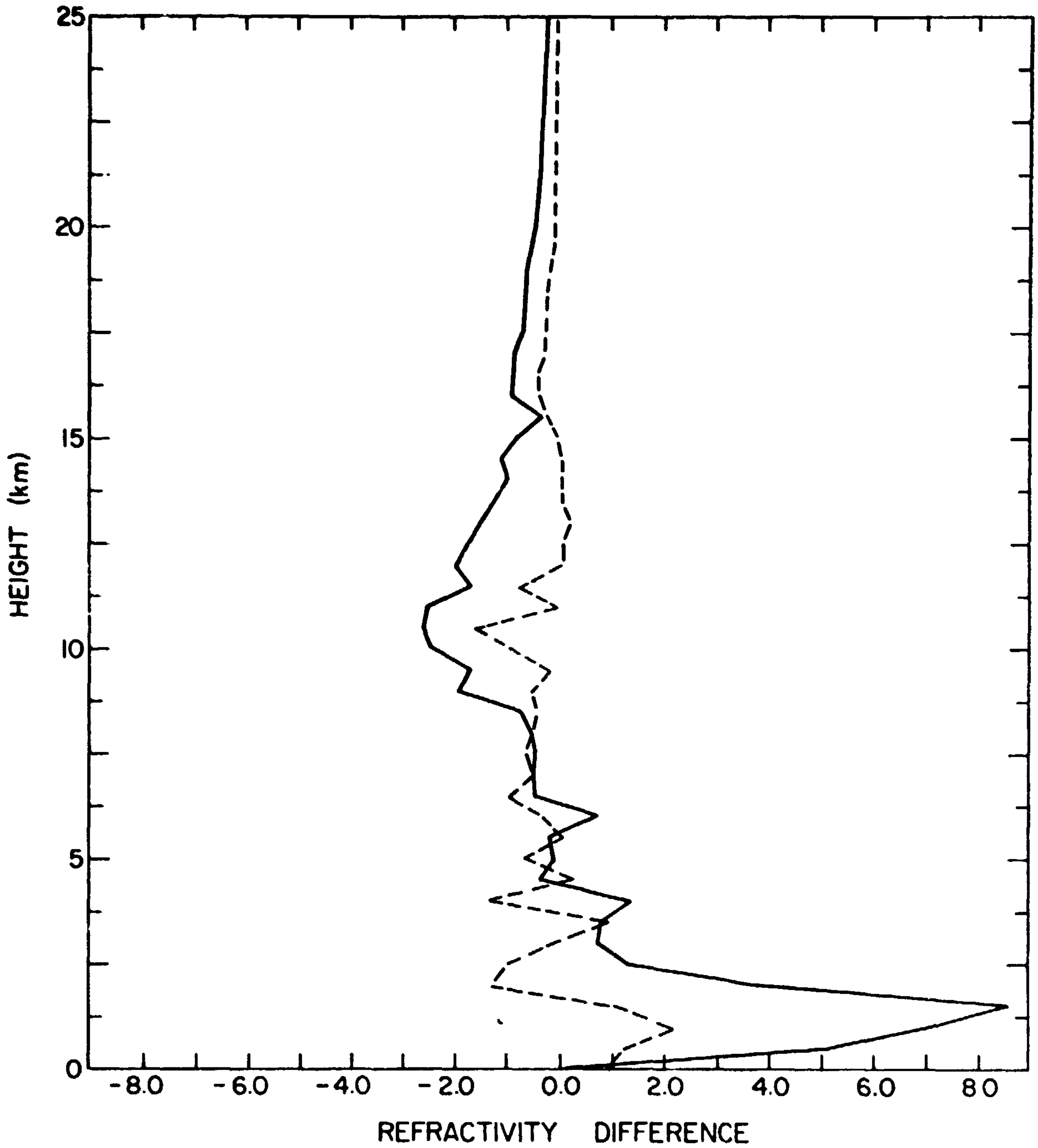

Figure 8. Difference between group refractivities measured along balloon trajectories versus height. Site 1 = 53. Site 2 = 55. Site 3 = 57. Time = 21:30. Date = 1/19/70. —— (N_{g1} - N_{g2}), - - - (N_{g1} - N_{g3}).

The difference between the refractivities directly above each site versus height is plotted in Figures 10 and 11. Figures 10 and 11 correspond to the same radiosonde balloon releases used to plot Figures 7 and 8. Notice that Figures 7 and 10 are similar. The reason for this is that since the refractivities along the three balloon trajectories (N_1, N_2, and N_3) differ by a small quantity, the coefficients N_θ and N_ϕ calculated by martix equation (3-4) are small. Therefore equation (3-3) for refractivity is essentially a function of height only (spherically symmetric). In this case, the refractivity calculated along the balloon trajectory at a height h is approximately equal to the refractivity calculated at the same height directly above the balloon release site. Figures 8 and 10 differ significantly since N_θ and N_ϕ are large (N_1, N_2, and N_3 differ by a large amount). It is interesting to note that the difference between a three-dimensional and spherically symmetric refractivity ray trace was larger when the balloon releases in Figure 8 (and 11) were used to generate the refractivity profiles.

The θ and ϕ gradients were also calculated directly above each release site using (3-7) and (3-8) and plotted in Figures 12 and 13 along with the group refractivity. If the refractivity profile was spherically symmetric these gradients would be zero. Equation (3-3) was used to calculate the refractivity. Note that both plots of group refractivity follow an exponential curve. The θ and ϕ gradients in Figure 13 have the same general tendencies, while the θ and ϕ gradients in Figure 12 are generally unrelated. In both figures, though the θ and ϕ gradients are not zero. This indicates that refractivity is not spherically symmetric. The question then is how do these asymmetries affect the accuracy of Marini and Murray's range correction formula?

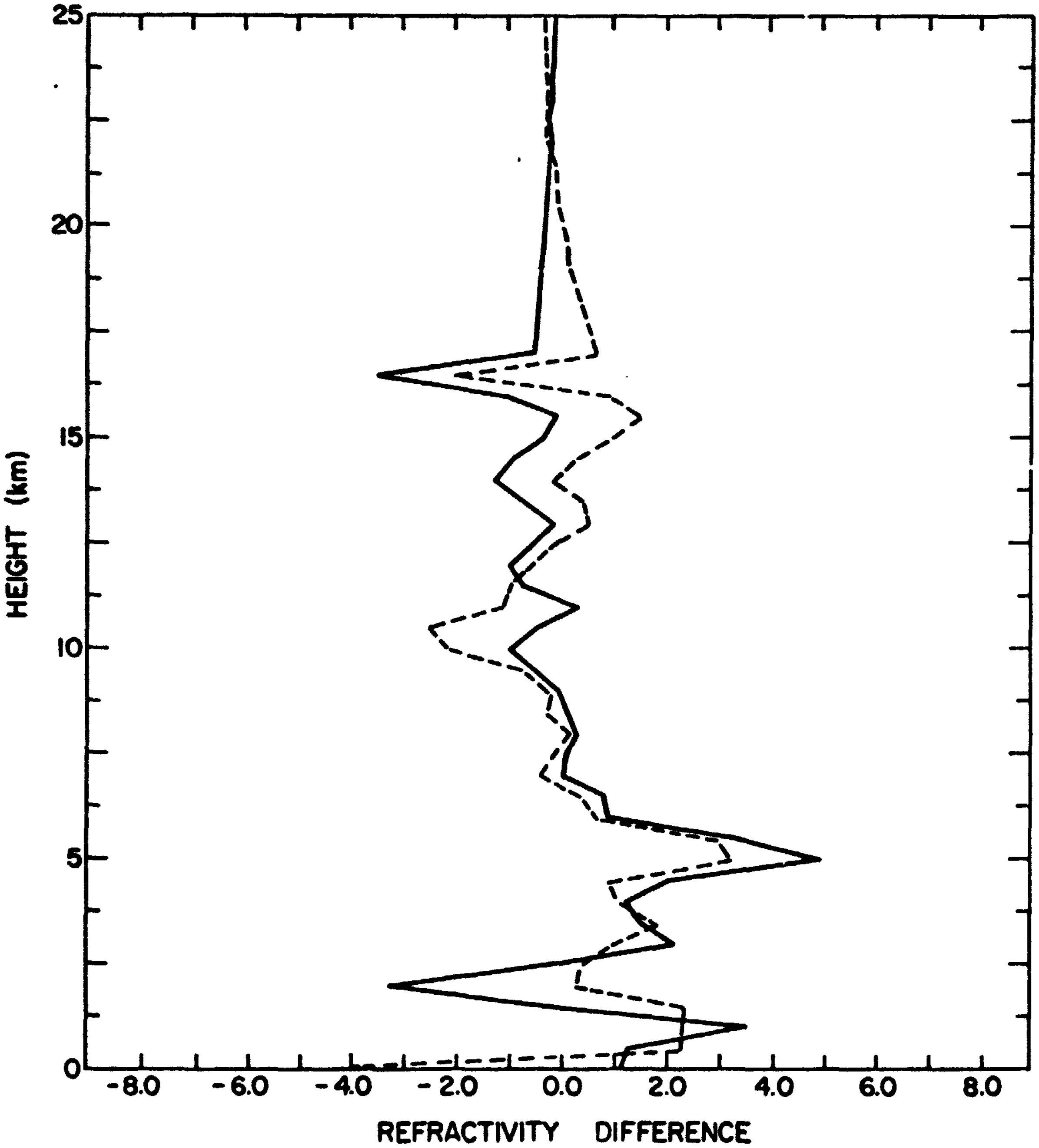

Figure 10. Difference between group refractivities calculated directly above each site versus height. Site 1 = 52. Site 2 = 55. Site 3 = 58. Time = 6:30. Date = 1/27/70. ——— $(N_{g1} - N_{g2})$, - - - $(N_{g1} - N_{g3})$.

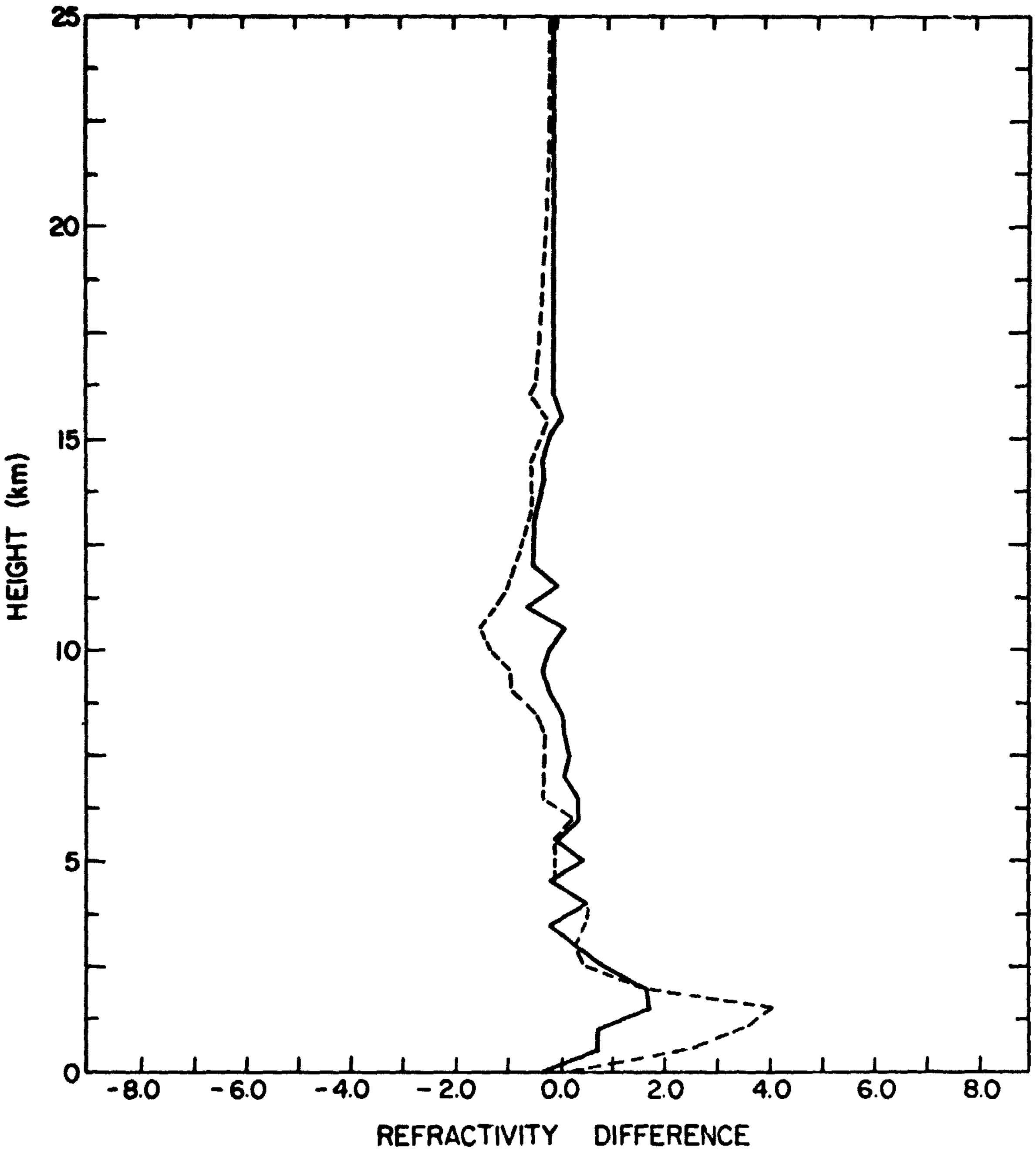

Figure 11. Difference between group refractivities calculated directly above each site versus height. Site 1 = 53. Site 2 = 55. Site 3 = 57. Time = 21:30. Date = 1/19/70. —— ($N_{g1} - N_{g2}$), - - - ($N_{g1} - N_{g3}$).

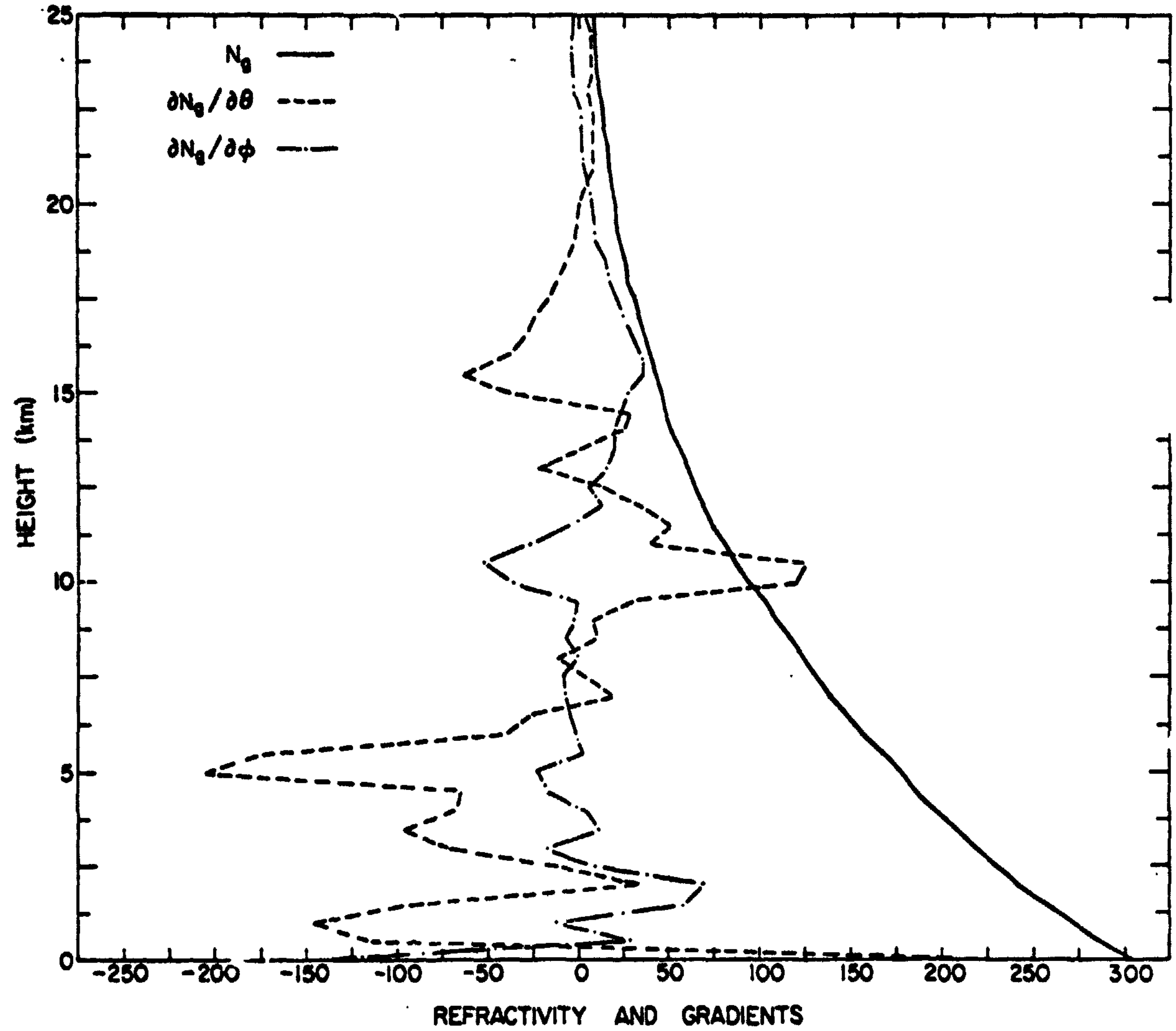

Figure 12. Group refractivities and partial derivatives with respect to θ and ϕ calculated directly above site 52. Auxiliary sites = 55 and 58. Time = 6:30. Date = 1/27/70. Partial derivatives (θ, ϕ) have a dimension of (radians)$^{-1}$.

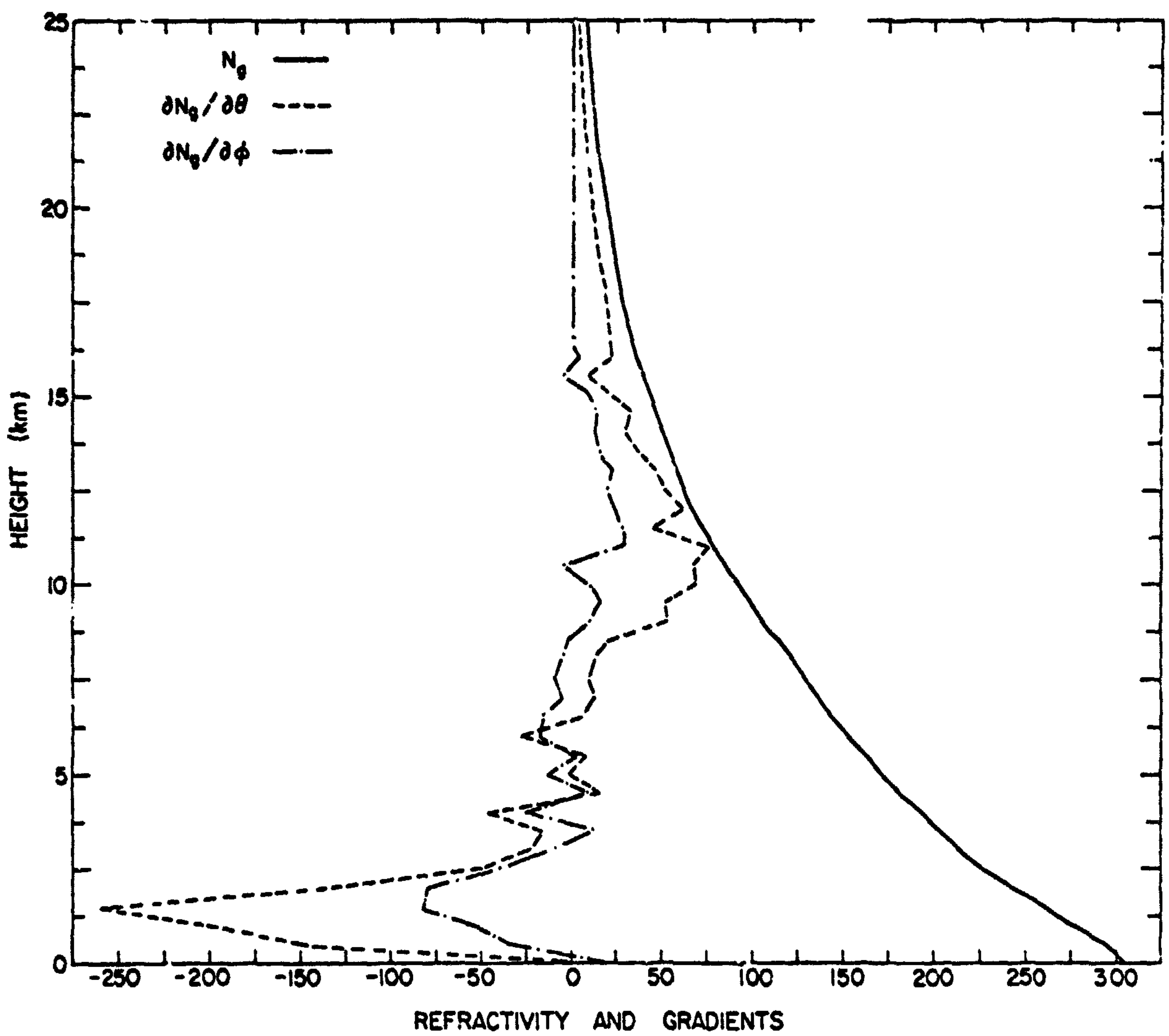

Figure 13. Group refractivities and partial derivatives with respect to θ and ϕ calculated directly above site 53. Auxiliary sites = 55 and 57. Time = 21:30. Date = 1/19/70. Partial derivatives (θ, ϕ) have a dimension of $(\text{radians})^{-1}$.

5. RESULTS FROM THREE-DIMENSIONAL REFRACTIVITY PROFILE RAY TRACING

The error introduced into laser ranging measurements by assuming the refractivity profile is spherically symmetric is investigated in this section. Ray traces through three-dimensional and spherically symmetric refractivity profiles were compared to evaluate this error. Appendix 1 gives the details of the ray tracing procedure. In creating the three-dimensional refractivity profile, radiosonde data from Project Haven Hop 1 was used. One balloon release site was assumed to be the ranging site. The remaining two release sites are located approximately 100 kilometers from the ranging site. Figure 14 illustrates the approximate locations of one set of three release sites. The three-dimensional ray trace, RT_3, was dependent on azimuth angle since the refractivity varied linearly in the

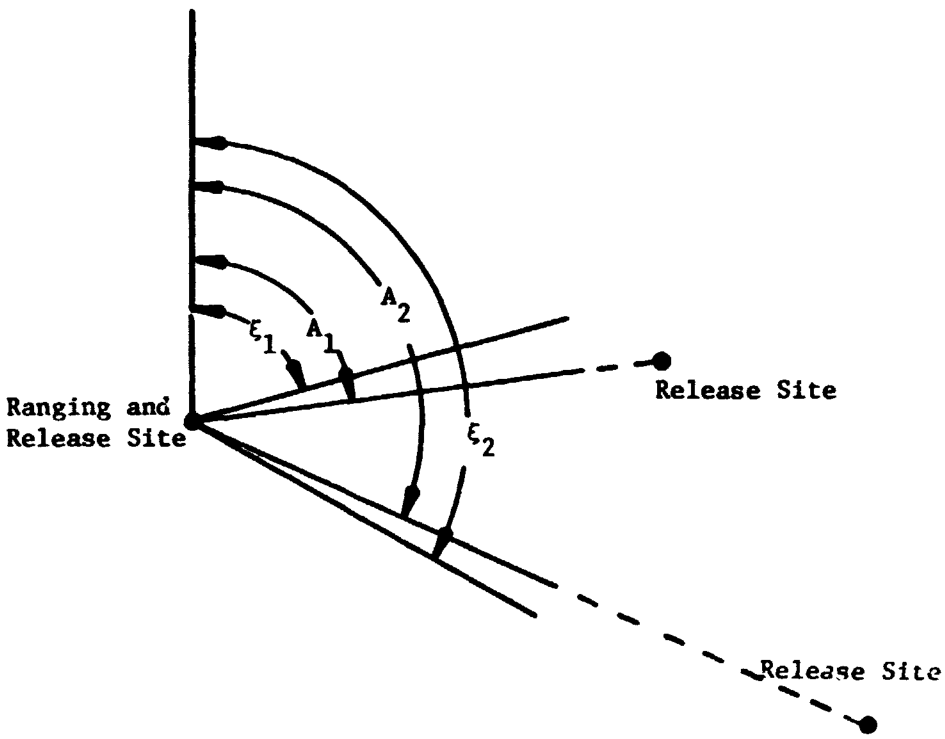

Figure 14. Locations of one set of release sites.

horizontal direction. For RT_3, the azimuth angles were selected in 10^{o} increments from the beginning azimuth, ξ_1, to the final azimuth, ξ_2. The azimuth angles formed by the laser site and remaining two release sites, A_1 and A_2, are used to calculate the beginning and final azimuth angles. ξ_1 is the closest azimuth angle less than A_1, which is an even multiple of 10^{o}. ξ_2 is the closest azimuth greater than A_2 which is an even multiple of 10^{o}. See Figure 14 for beginning and final azimuth angles for one set of three balloon release sites. The spherically symmetric refractivity profile was calculated from the balloon released from the ranging site. RT_1 was independent of azimuth since refractivity was only a function of height. Each spherically symmetric ray trace, therefore, was compared to three-dimensional ray traces at different azimuth angles. Figures 15 through 21 represent graphs of this comparison. The data points (RT_3-RT_1), which were calculated every 10^{o}, were connected by straight lines. In Figure 3, the balloon released from the ranging site was blown at an azimuth angle of approximately 90^{o}. When the same balloon releases as illustrated in Figure 3 were used, the difference betwee RT_3-RT_1 was small at an azimuth of 90^{o} (See Figure 15 for these results). This is expected since the θ and ϕ along the ray path are similar to the θ and ϕ of the balloon trajectory at a given height.

The histograms in Figures 21, 22, 23, and 24 were obtained from 428 comparisons of ray traces through three dimensional and spherically symmetric refractivity profiles. The 428 ray traces at each elevation angle were obtained from 17 different sets of 3 balloon releases. For each set of releases, rays were traced at 8 to 9 azimuth angles from each of the three release sites. All refractivity profiles were generated for a wavelength of 530 nanometers, which is the wavelength for the doubled Yag laser. The actual data for these 428 ray traces is in Appendix B pages 67 to 134. It is interesting to compare the results obtained in this investigation to Marini and Murray's results (Table 1). At an elevation angle of 10^{o} the standard deviation of RT_3-RT_1 is 287 centimeters. Marini and Murray calculated the standard deviation of the difference between their formula and

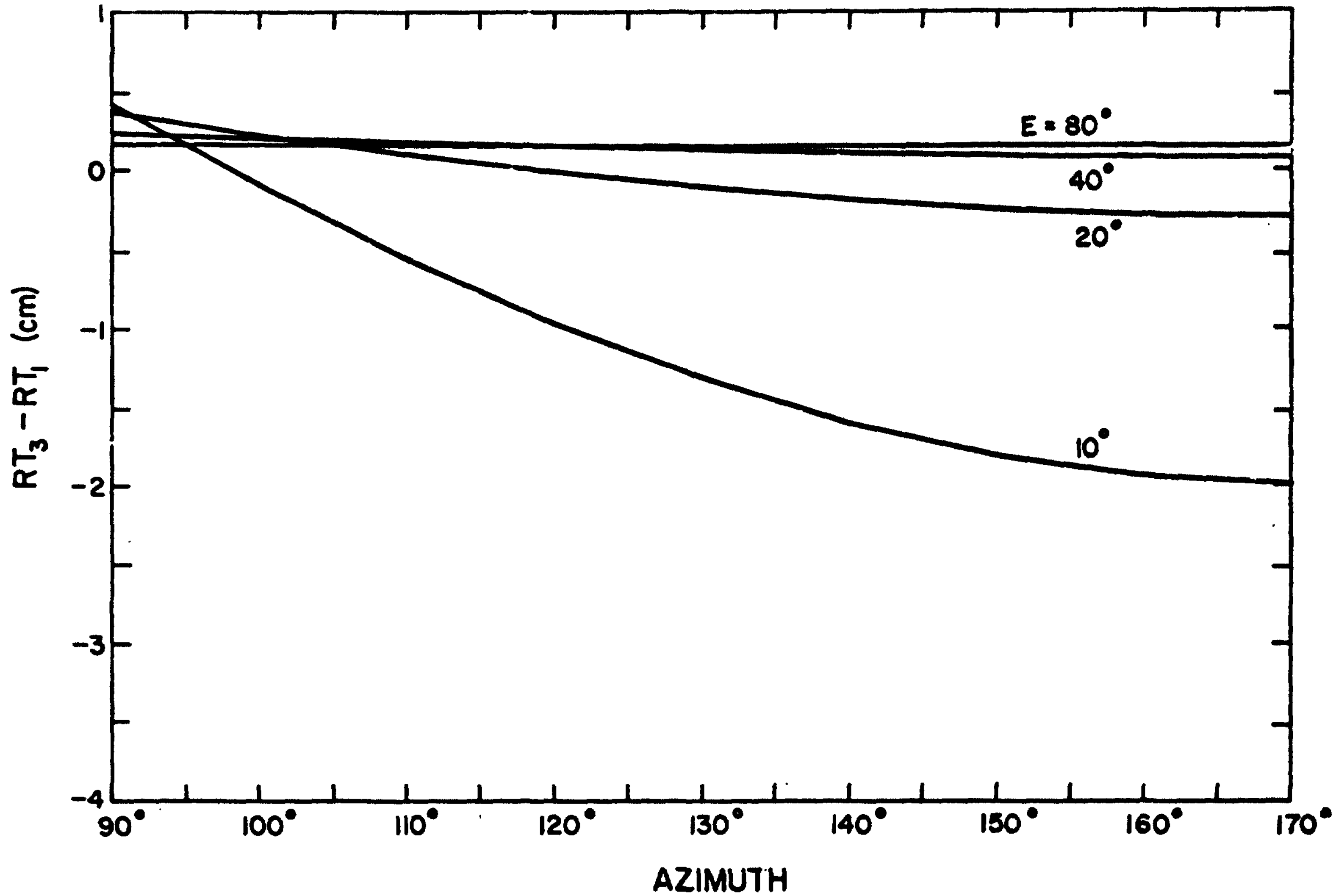

Figure 15. **Difference between three-dimensional refractivity ray trace (RT_3) and spherically symmetric refractivity ray trace (RT_1) versus azimuth. Laser site = 52. Aux. sites = 53 and 56. Time = 17:30. Date = 2/13/70.**

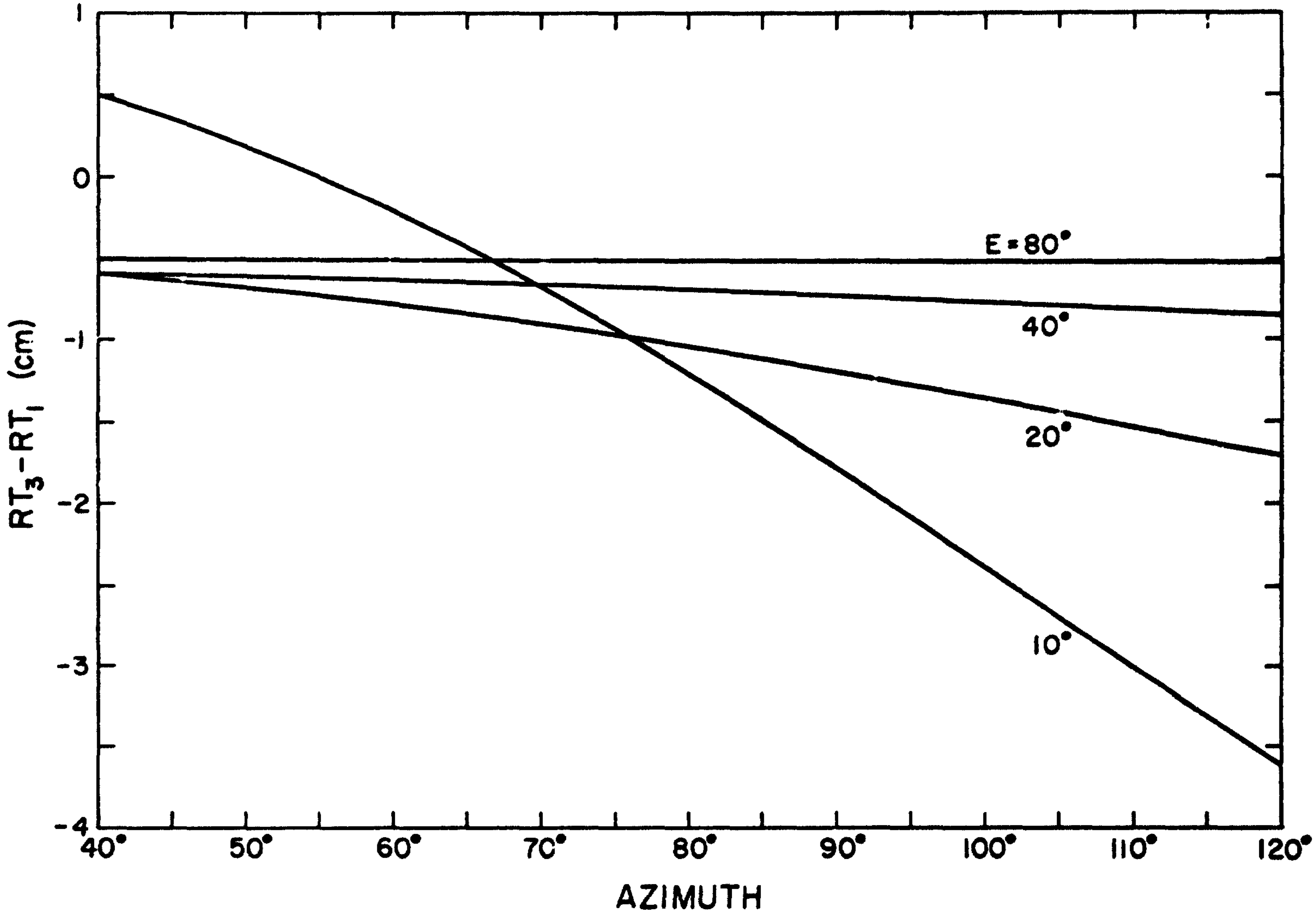

Figure 16. Difference between three-dimensional refractivity ray trace (RT_3) and spherically symmetric refractivity ray trace (RT_1) versus azimuth. Laser site = 55. Aux. sites = 53 and 57. Time = 21:30. Date = 1/19/70.

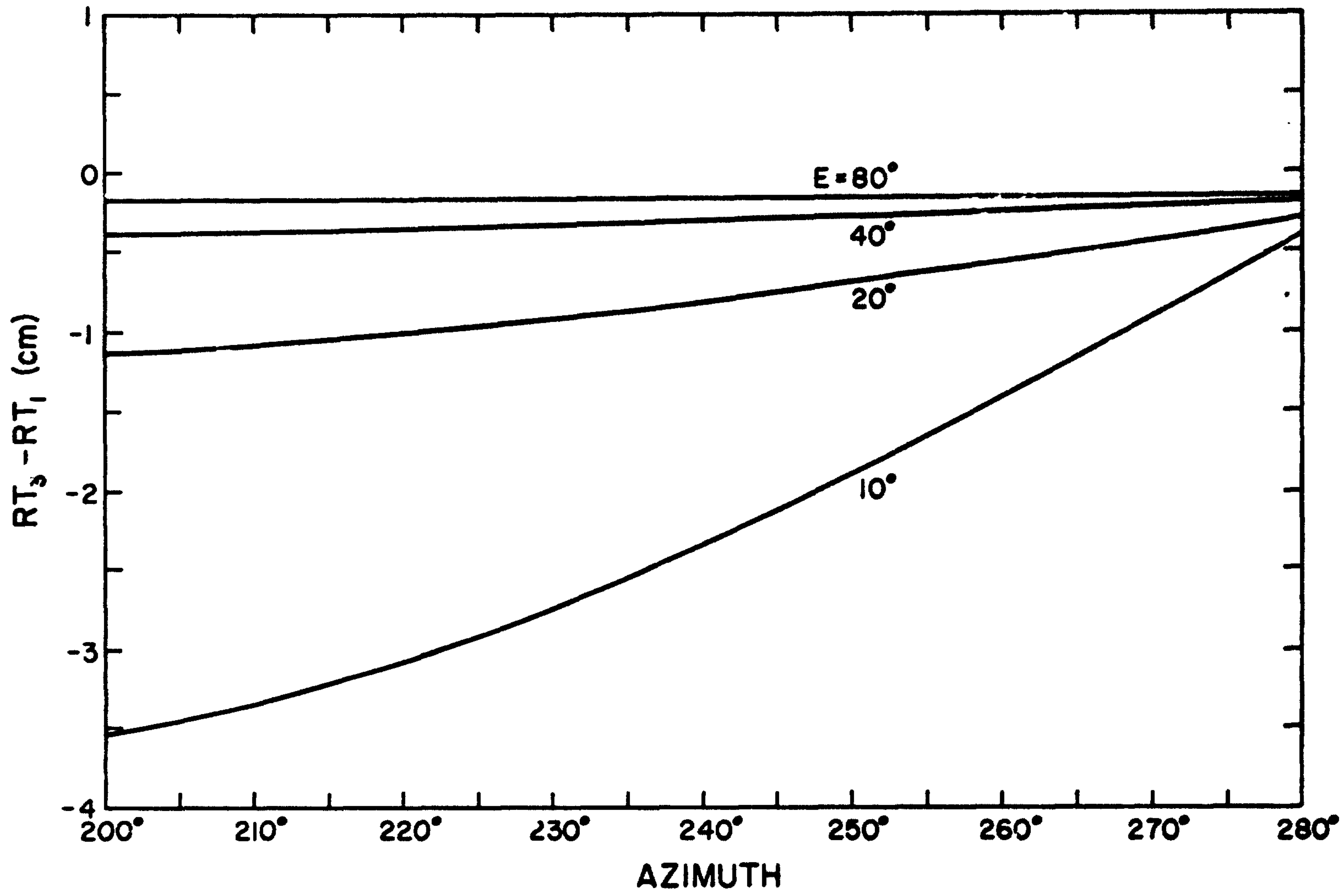

Figure 17. Difference between three-dimensional refractivity ray trace (RT_3) and spherically symmetric refractivity ray trace (RT_1) versus azimuth. Laser site = 53. Aux. sites = 52 and 56. Time = 17:30. Date = 2/13/70.

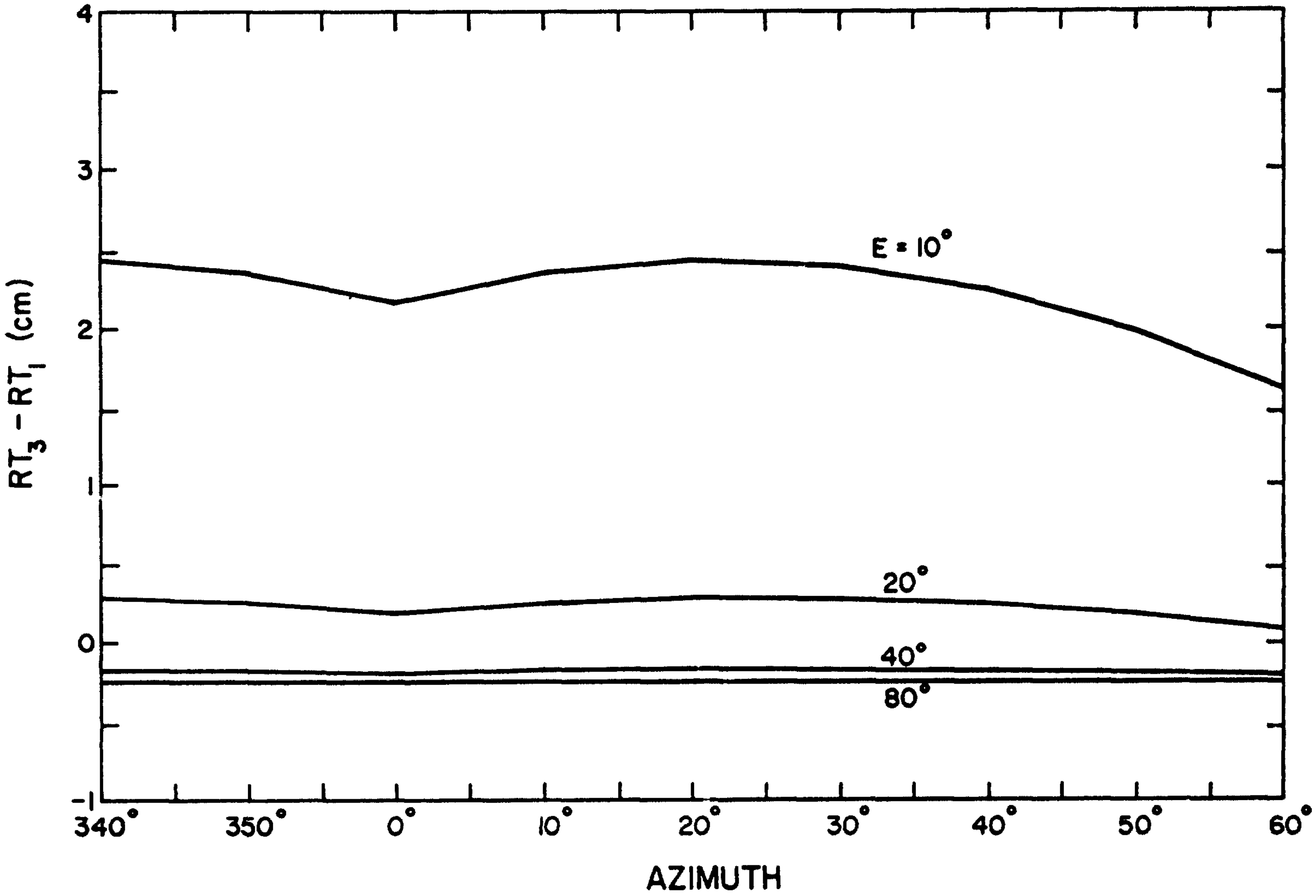

Figure 18. Difference between three-dimensional refractivity ray trace (RT_3) and spherically symmetric refractivity ray trace (RT_1) versus azimuth. Laser site = 56. Aux. sites = 52 and 53. Time = 19:30. Date = 2/13/70.

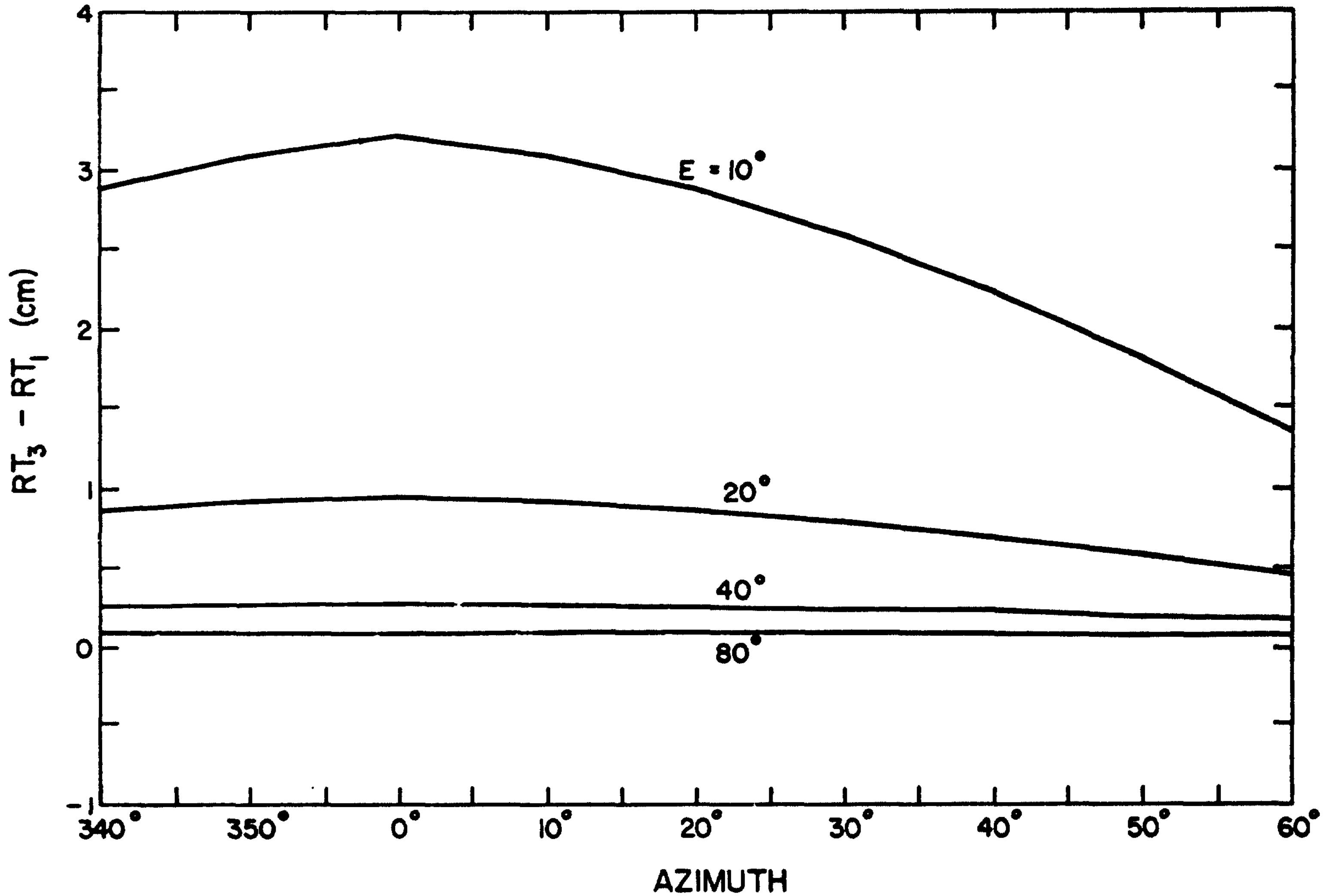

Figure 19. Difference between three-dimensional refractivity ray trace (RT_3) and spherically symmetric refractivity ray trace (RT_1) versus azimuth. Laser site = 56. Aux. sites = 52 and 53. Time = 17:30. Date = 2/13/70.

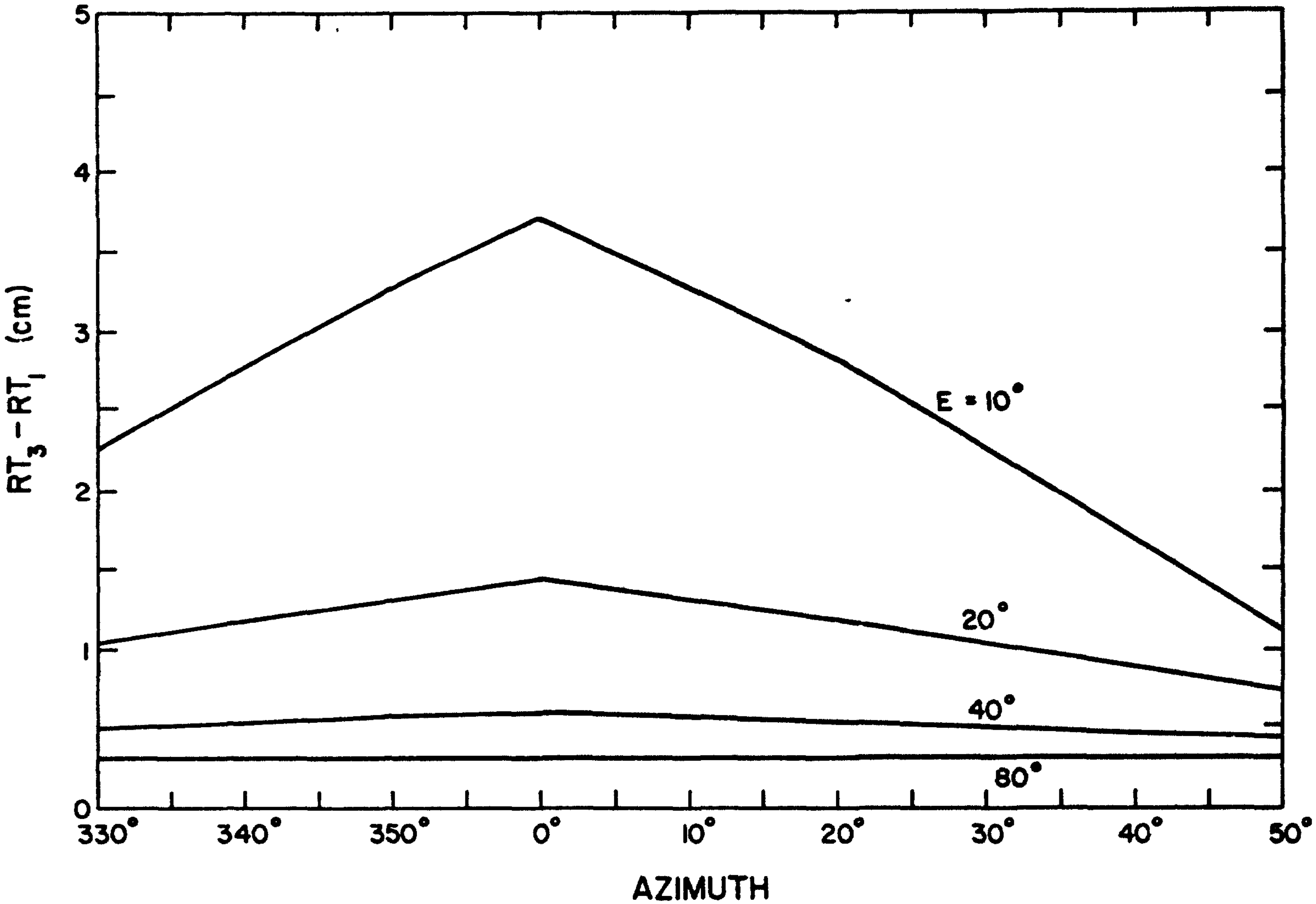

Figure 20. Difference between three-dimensional refractivity ray trace (RT_3) and spherically symmetric refractivity ray trace (RT_1) versus azimuth. Laser site = 56. Aux. sites = 52 and 54. Time = 17:30. Date = 1/15/70.

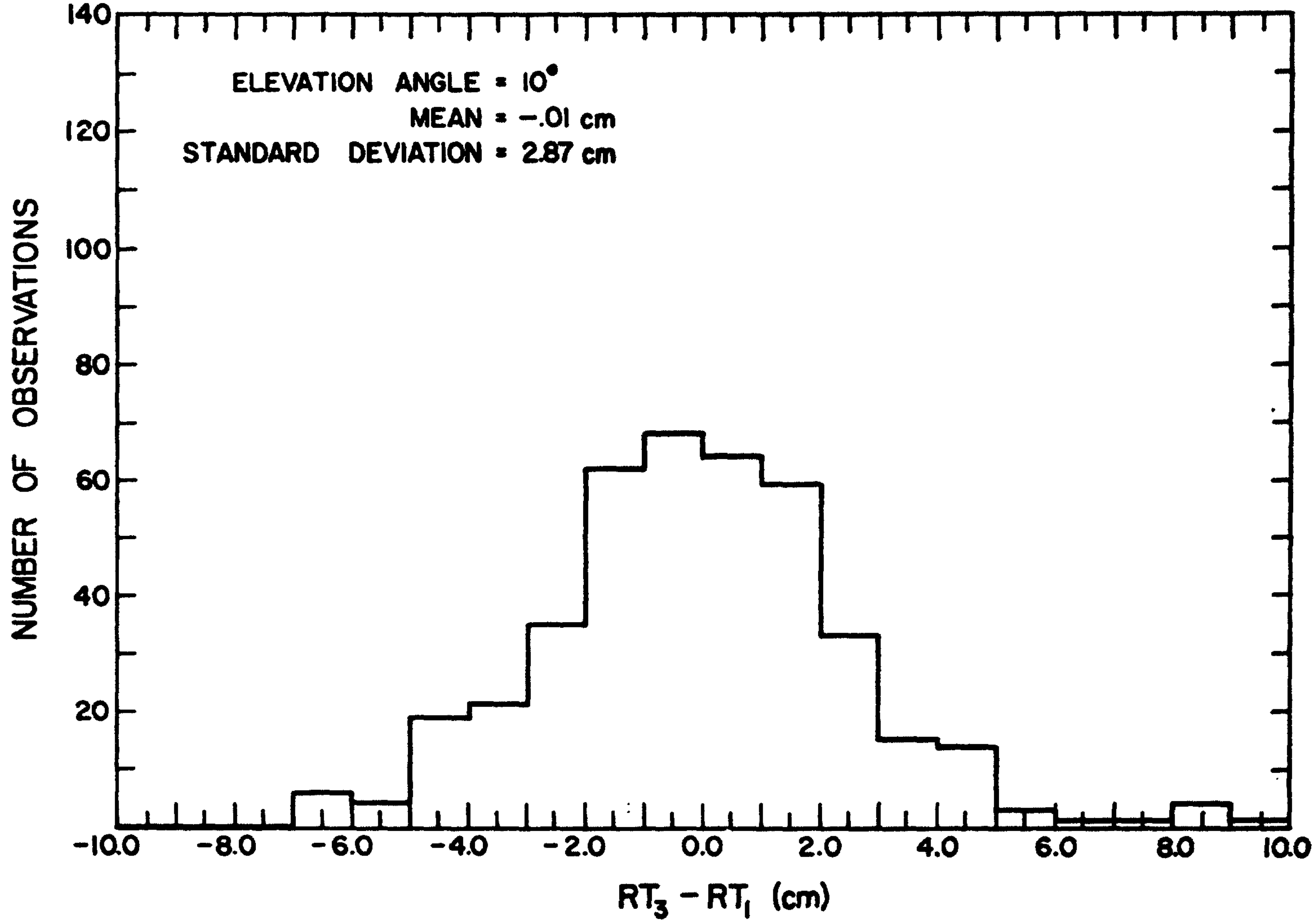

Figure 21. Histogram of RT_3-RT_1 data for E = 10° and wavelength = 530 nanometers. Sample size = 428.

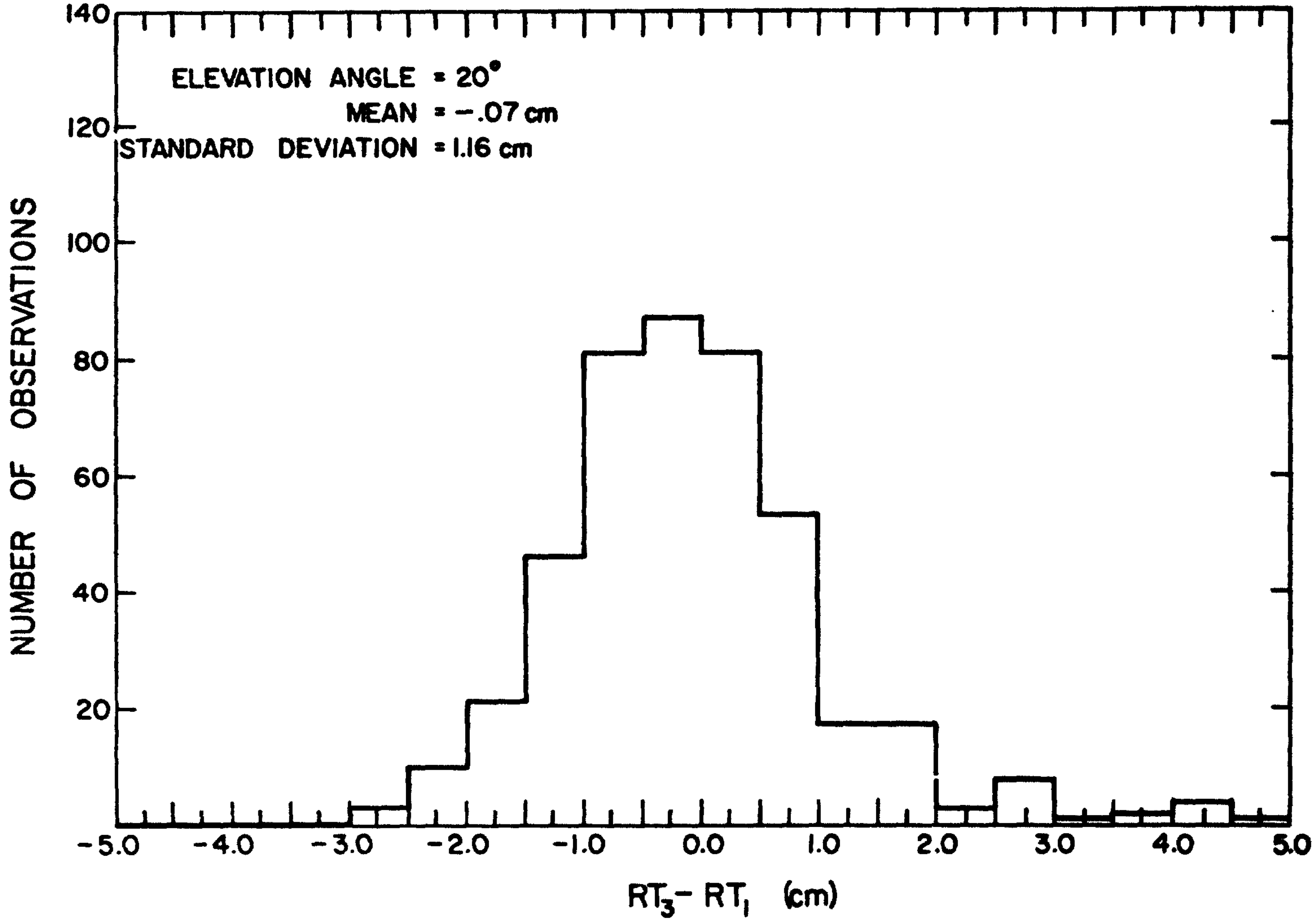

Figure 22. Histogram of RT_3-RT_1 data for E = 20° and wavelength = 530 nanometers. Sample size = 428.

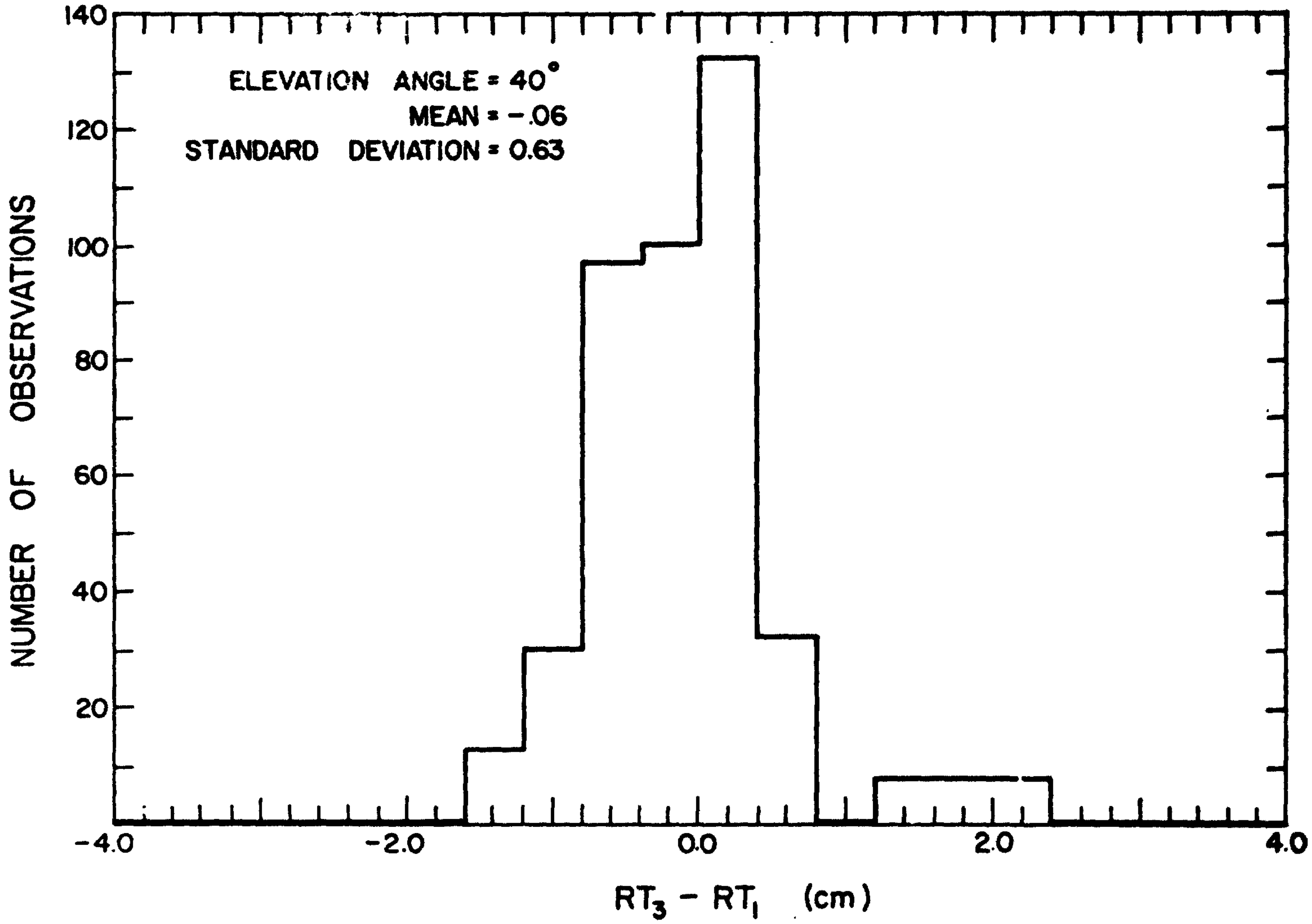

Figure 23. Histogram of RT_3-RT_1 data for E = 40° and wavelength = 530 nanometers. Sample size = 428.

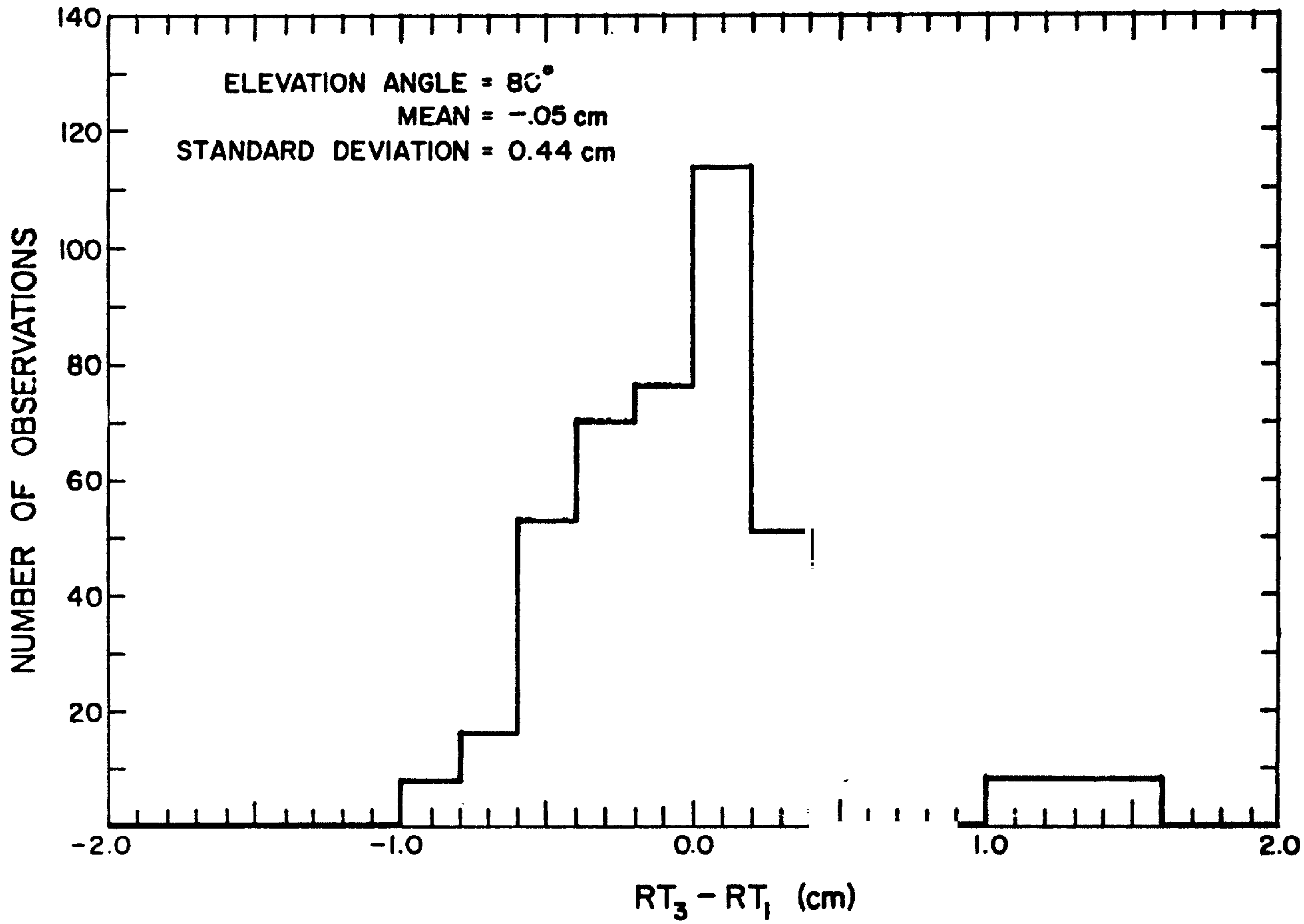

Figure 24. Histogram of RT3-RT1 data for E = 80° and wavelength = 530 nanometers. Sample size = 428.

ray tracing through a spherically symmetric profile (RT_1-MM) to be only .49 centimeters at E = 10^o.The standard deviations of RT_3-RT_1 obtained in this investigation are larger than Marini and Murray's standard deviations of RT_1-MM by almost a factor of 10 at elevation angles of 20^o, 40^o, and 80^o. The distributions in Figures 25, 26, 27, and 28 were calculated from the same data used for the histograms (N = 428). The percentage of RT_3-RT_1 values greater than a value can be found from these distributions. The dashed line represents a theoretical Gaussian distribution with zero mean and standard deviation equal to the standard deviation calculated at each elevation angle. Notice that 84% of the RT_3-RT_1 values at E = 10^o were greater than Marini and Murray's standard deviation at 10^o. The other elevation angles calculated also had high percentages greater than Marini and Murray's standard deviation (85% @ E = 20^o, 85% @ E = 40^o, 91% @ E = 80^o).

The spherically symmetric refractivity ray traces which we evaluated were also compared to Marini and Murray's formula, MM. Table 2 summarizes these results along with the results obtained for RT_3-RT_1 and RT_3-MM. For our data the mean values of RT_1-MM at different elevation angles are much larger than Marini and Murray's data (summarized in Table 1). We believe this can be partially accounted for by the fact that our radiosonde data was taken only during January and February of 1970 and in only one locality, the Washington, D. C. area. Marini and Murray used data from more than one locality and over longer periods of time [3]. The standard deviations between RT_1 and MM compare very well with Marini and Murray's results. Marini and Murray's results were obtained for a ruby laser at a wavelength of 694 nanometers. To obtain the mean between RT_3 and MM the means of RT_3-RT_1 and RT_1-MM can just be added. The standard deviation of RT_3-MM can be found from the standard deviations of RT_3-RT_1 and RT_1-MM by assuming these values are

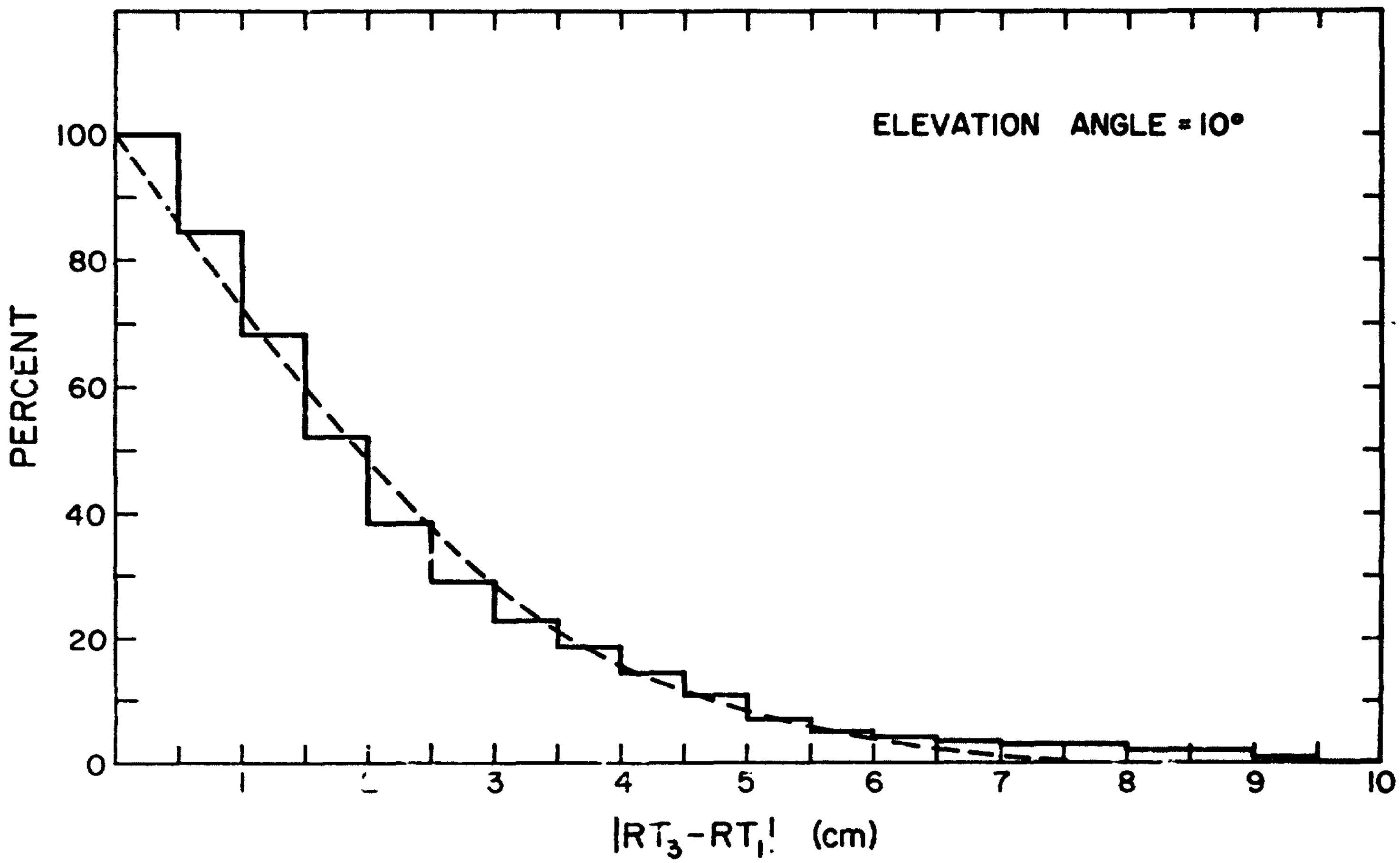

Figure 25. Distribution of RT_3-RT_1 data for E = 10° and wavelength = 530 nanometers. Sample size = 428. (- - -) represents Gaussian distribution of zero mean and standard deviation = 2.87 centimeters.

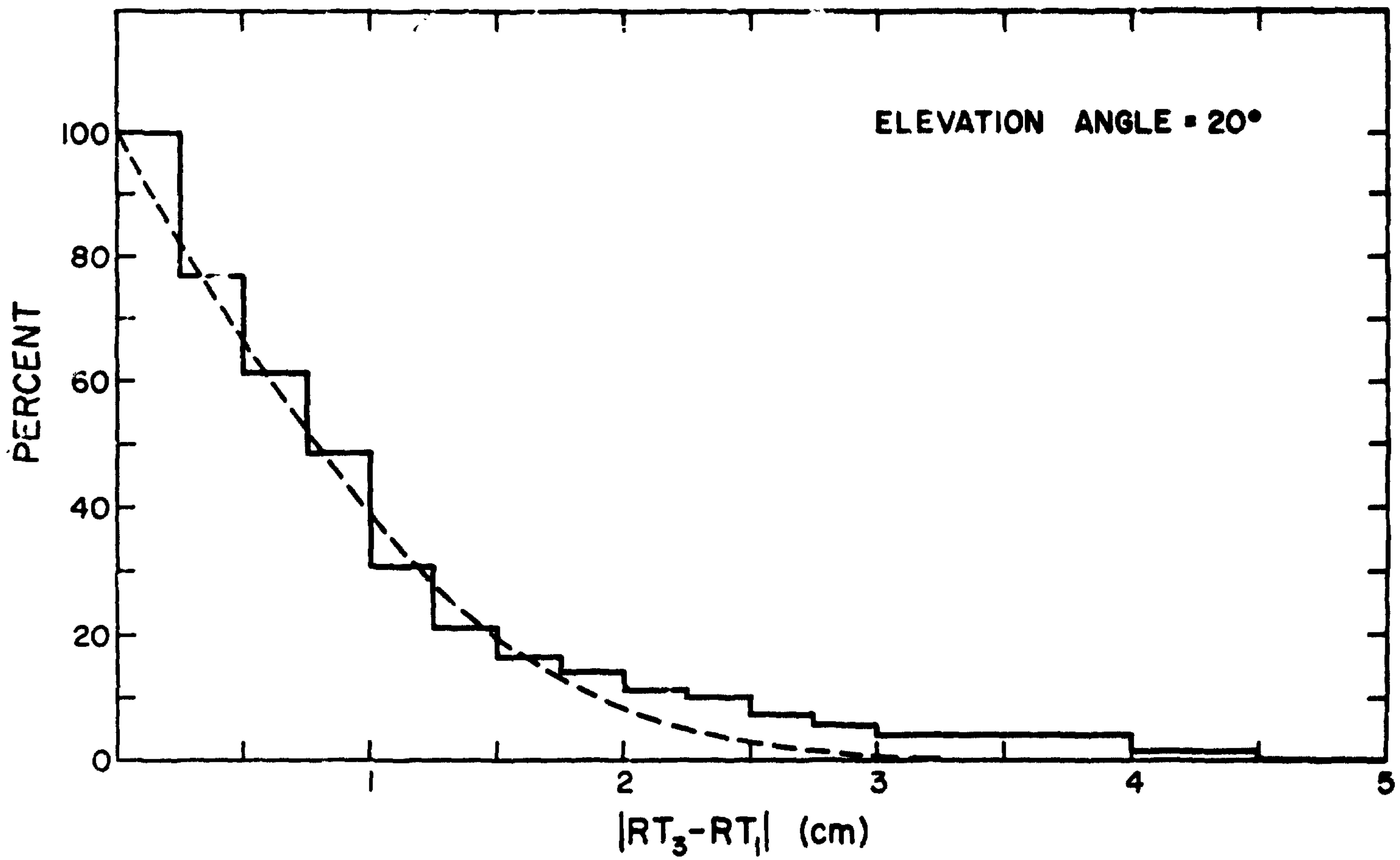

Figure 26. Distribution of RT_3-RT_1 data for E = 20° and wavelength = 530 nanometers. Sample size = 428. (- - -) represents Gaussian distribution of zero mean and standard deviation = 1.16 centimeters.

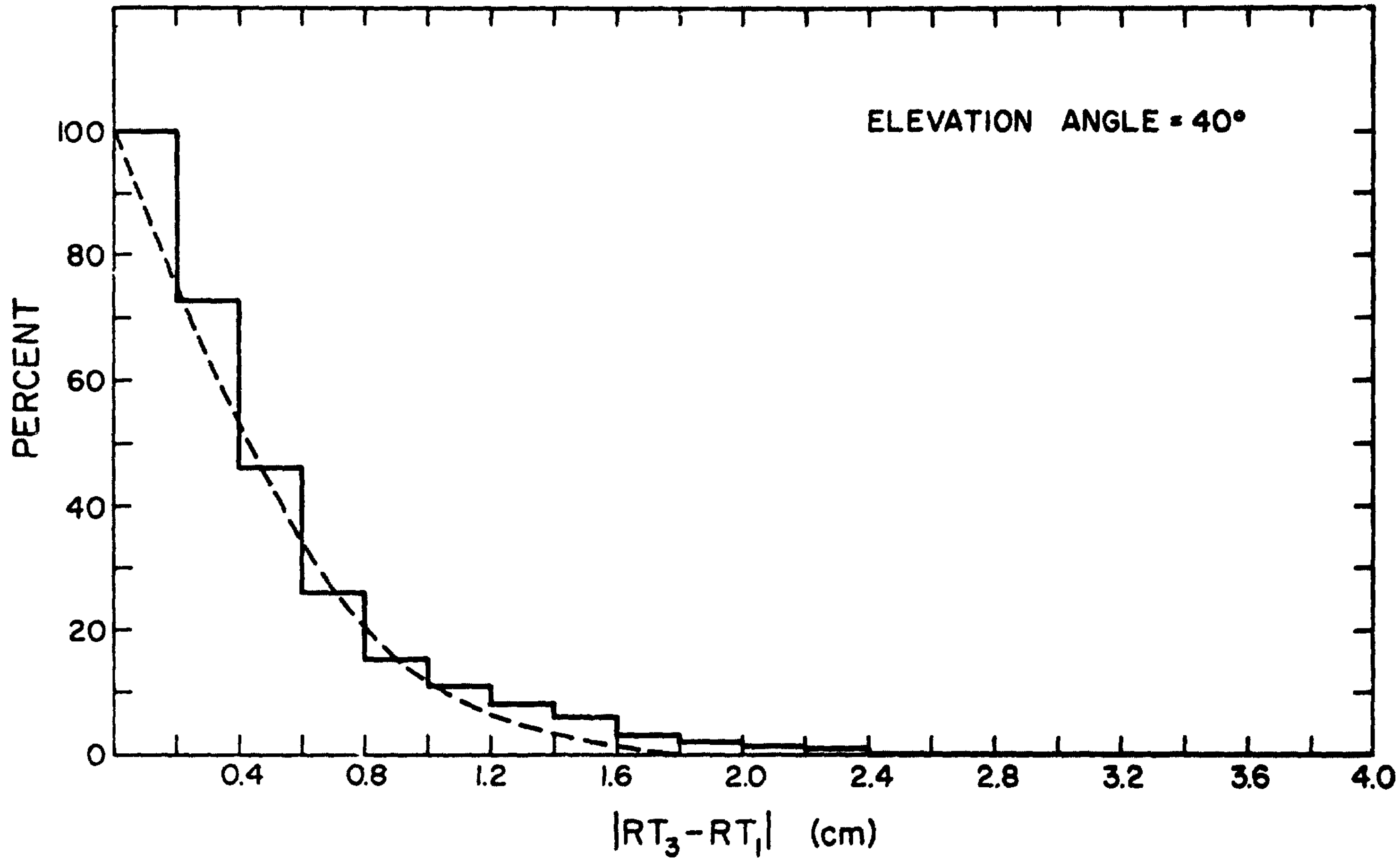

Figure 27. Distribution of RT_3-RT_1 data for E = 40° and wavelength = 530 nanometers. Sample size = 428. (- - -) represents Gaussian distribution of zero mean and standard deviation = .63 centimeters.

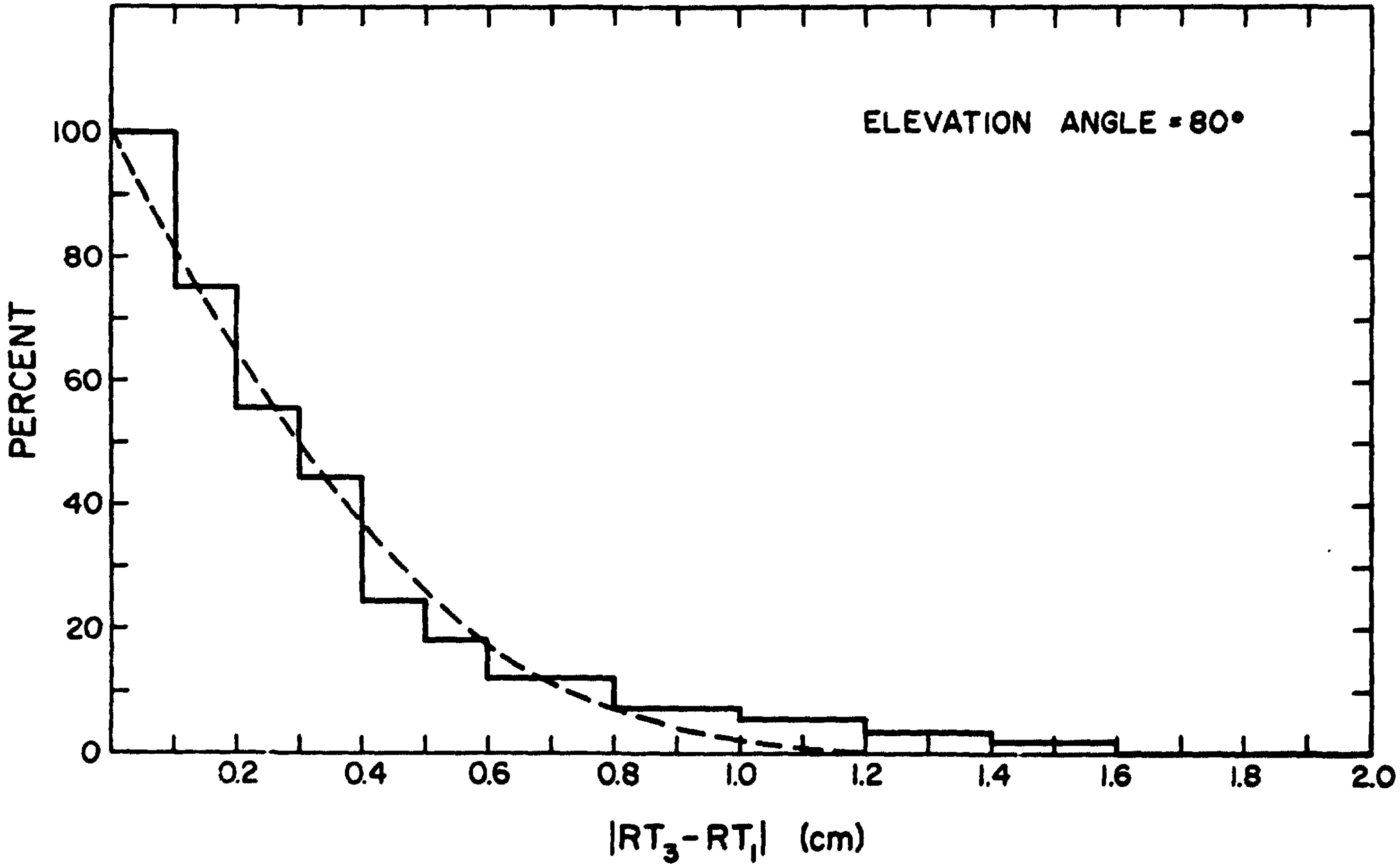

Figure 23. Distribution of RT_3-RT_1 data for E = 80° and wavelength = 530 nanometers. Sample size = 428. (- - -) represents Gaussian distribution of zero mean and standard deviation = .44 centimeters.

TABLE 2

SUMMARY OF RT_1-MM, RT_3-RT_1, AND RT_3-MM RESULTS

	RT_1-MM N=51		RT_3-RT_1 N=428		RT_3-MM N=428	
E	Mean (cm)	Standard Deviation (cm)	Mean (cm)	Standard Deviation (cm)	Mean (cm)	Standard Deviation (cm)
80	.10	.06	-.05	.44	.05	.44
40	.15	.09	-.06	.63	.09	.64
20	.37	.18	-.07	1.16	.30	1.17
10	.69	.49	-.01	2.87	.68	2.91

λ = 530 nanometers

statistically indepe. Using the abbreviation STD for standard deviation equation (5-1) results.

$$STD(RT_3-MM) = (STD^2(RT_3-RT_1) + STD^2(RT_1-MM))^{\frac{1}{2}} \qquad (5\text{-}1)$$

Equation (5-1) was used to calculate standard deviations of RT_3-MM in Table 2. We believe that Marini and Murray's results are valid when the refractivity is spherically symmetric. Saastamoinen [1] estimated that the departure of the atmosphere for spherical symmetry would introduce at most 1 to 2 centimeters error into the range correction formula at 10^o elevation. However our data for 10^o elevation (Figure 26) indicates that 68% of the values (errors introduced by nonsymmetrical refractivity profile) are greater than 1 centimeter and 38% are greater than 2 centimeters.

The dependence of RT_3-RT_1 on wavelength was examined by comparing a small sample (N=42) of ray traces at 530 and 353 nanometers. The wavelength for the tripled Yag laser is 353 nanometers. The data obtained from this wavelength comparison is in Appendix B, pages 135 to 144. Notice from equation (2-3)

that group refractivity is inversely related to wavelength. Figures 29, 30, 31, 32, and 33 are plots of this comparison. The difference between the two frequencies is small when RT_3-RT_1 is small (Figure 33) and l[illegible] when RT_3-RT_1 is large (Figure 30). Notice though, that the value of $|RT_3 - RT_1|$ at a lower wavelength is always greater than at a higher wavelength. This is expected since the refractivity increases with decreasing wavelength (equation 2-3). Figure 33 gives a good illustration of this fact. The results of frequency dependence are summarized in Table 3.

TABLE 3

WAVELENGTH COMPARISON OF RT_3-RT_1

	$RT_3 - RT_1$ N = 42 λ = 353 nanometers		$RT_3 - RT_1$ N = 42 λ = 530 nanometers	
E	Mean (cm)	Standard Deviation (cm)	Mean (cm)	Standard Deviation (cm)
80	-.25	.42	-.23	.39
40	-.22	.63	.20	.58
20	.02	1.20	.02	1.12
10	1.32	2.55	1.23	2.35

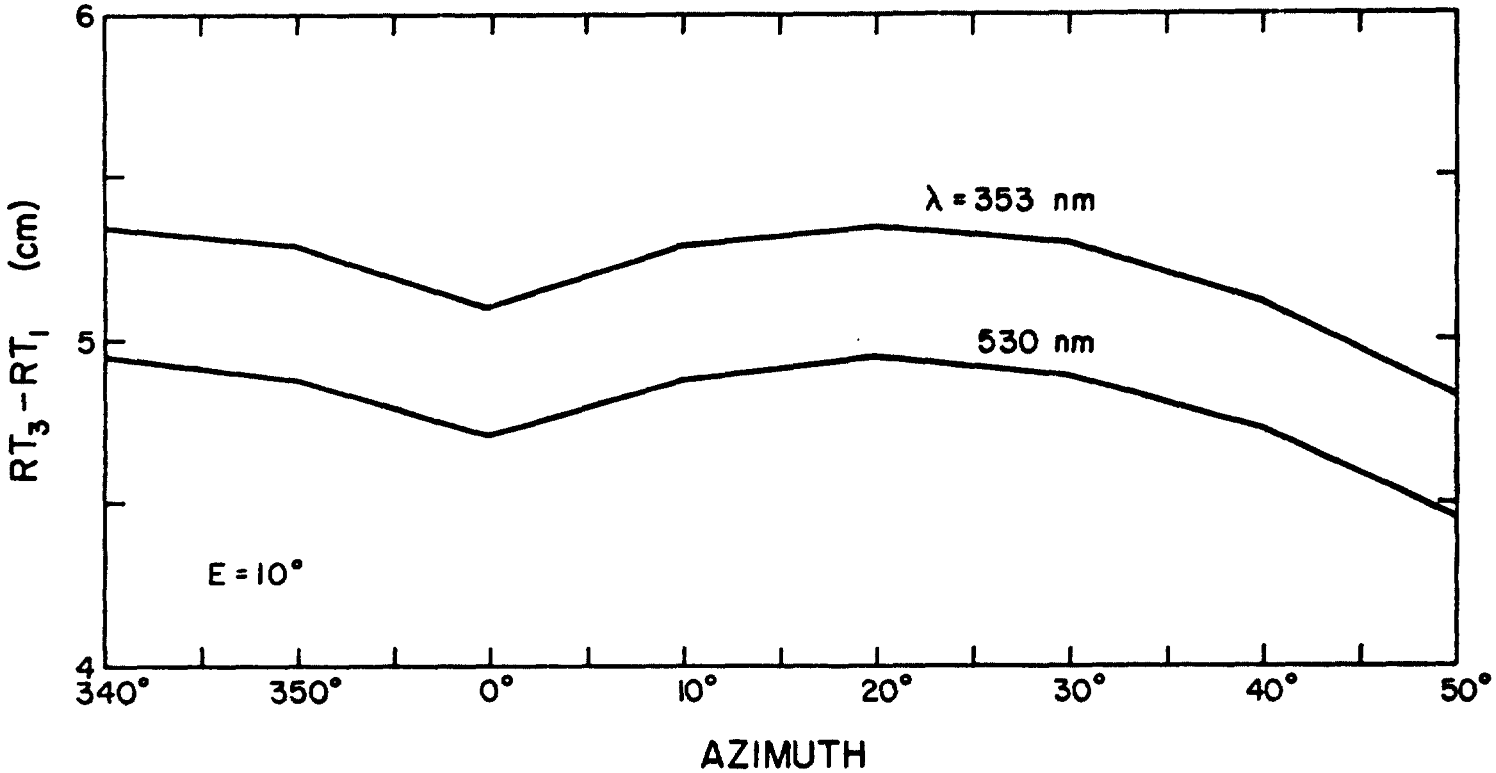

Figure 29. Comparison of RT_3-RT_1 at λ = 353 and 530 nanometers. E = 10°. Laser site = 56. Aux. sites = 52 and 54. Time = 23:30. Date = 1/21/70.

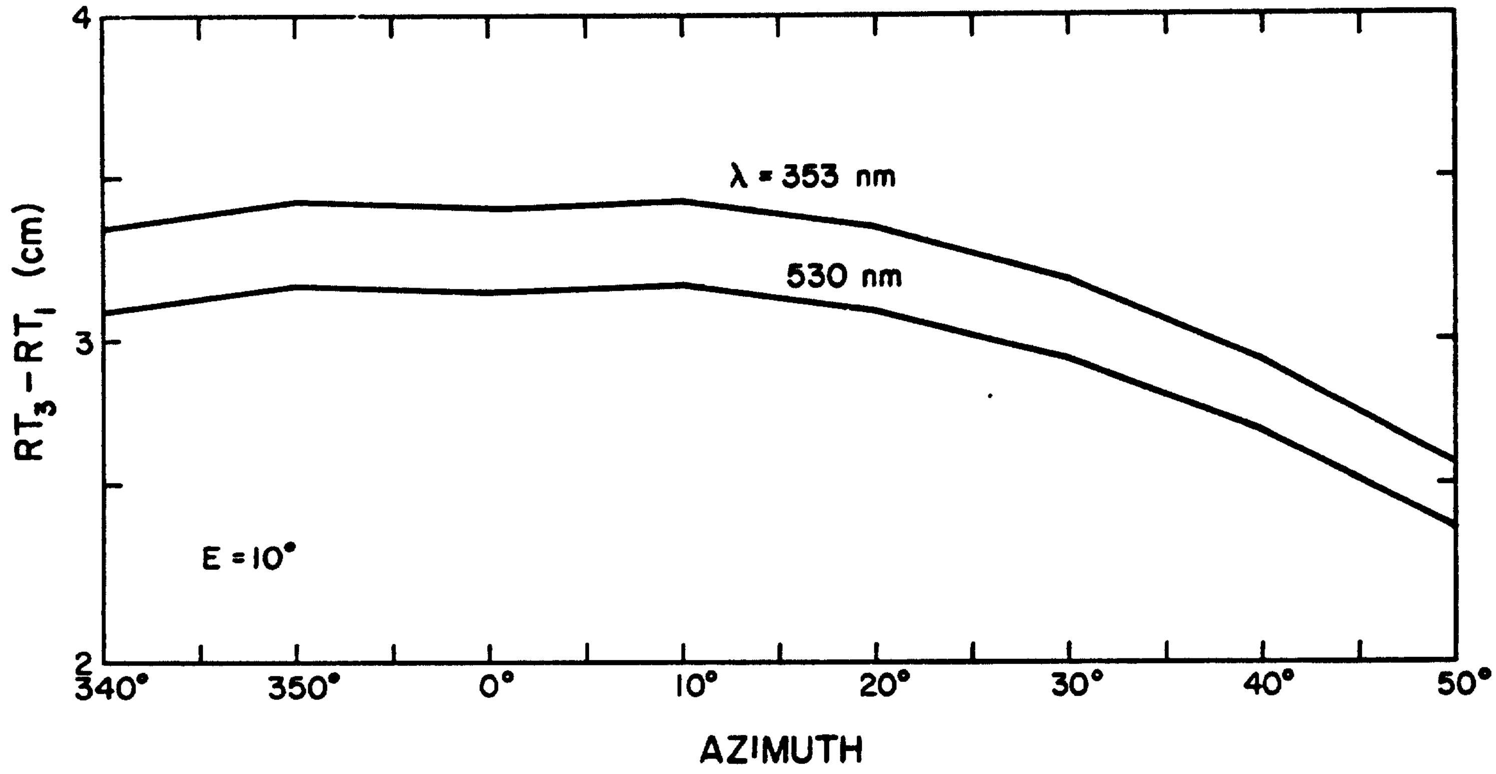

Figure 30. Comparison of RT_3-RT_1 at λ = 353 and 530 nanometers.
E = 10°. Laser site = 56. Aux. sites = 52 and 55.
Time = 21:30. Date = 1/21/70.

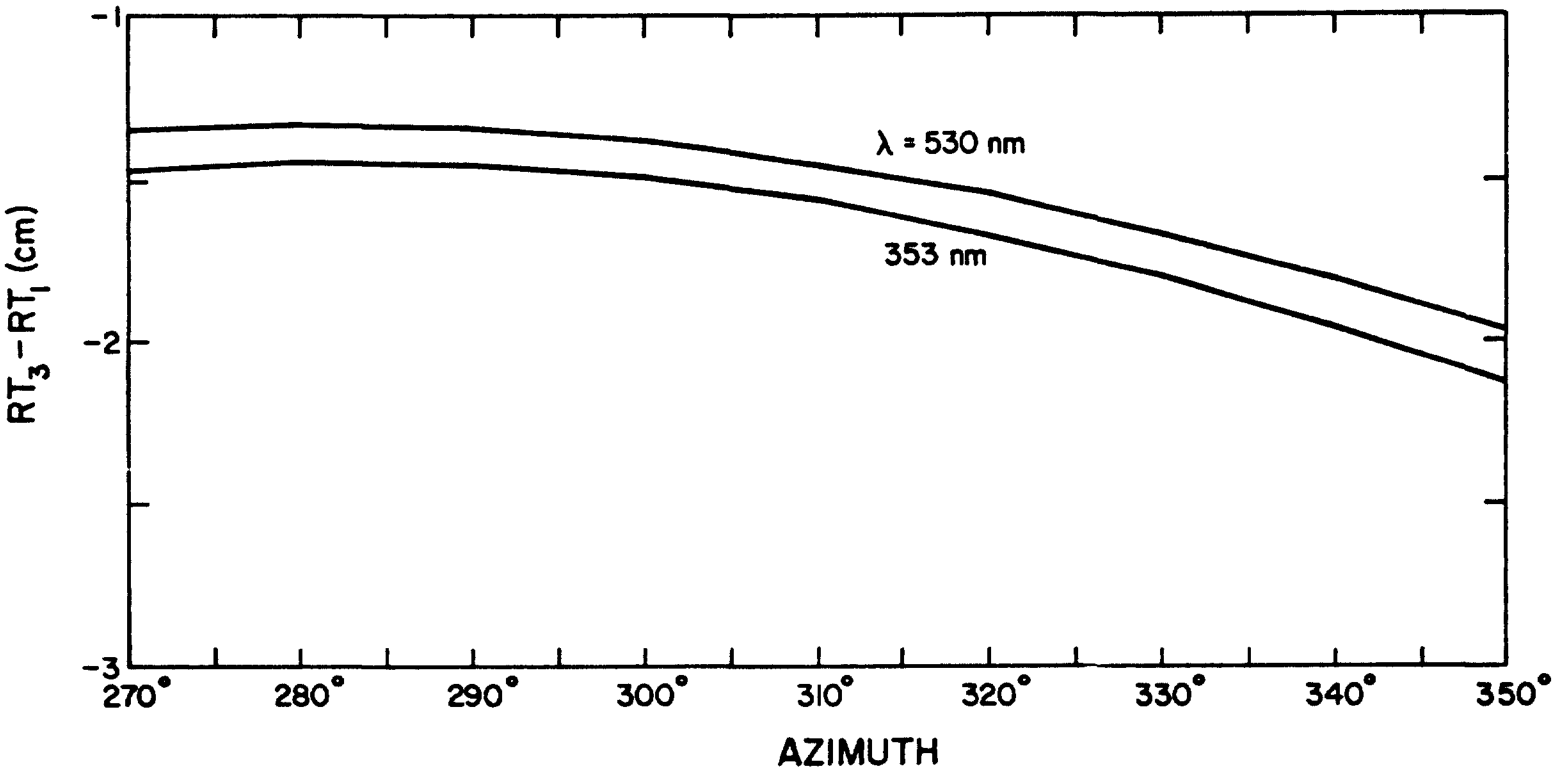

Figure 31. Comparison of RT_3-RT_1 at λ = 353 and 530 nanometers.
E = 10°. Laser site = 58. Aux. sites = 52 and 55.
Time = 17:30. Date = 1/19/70.

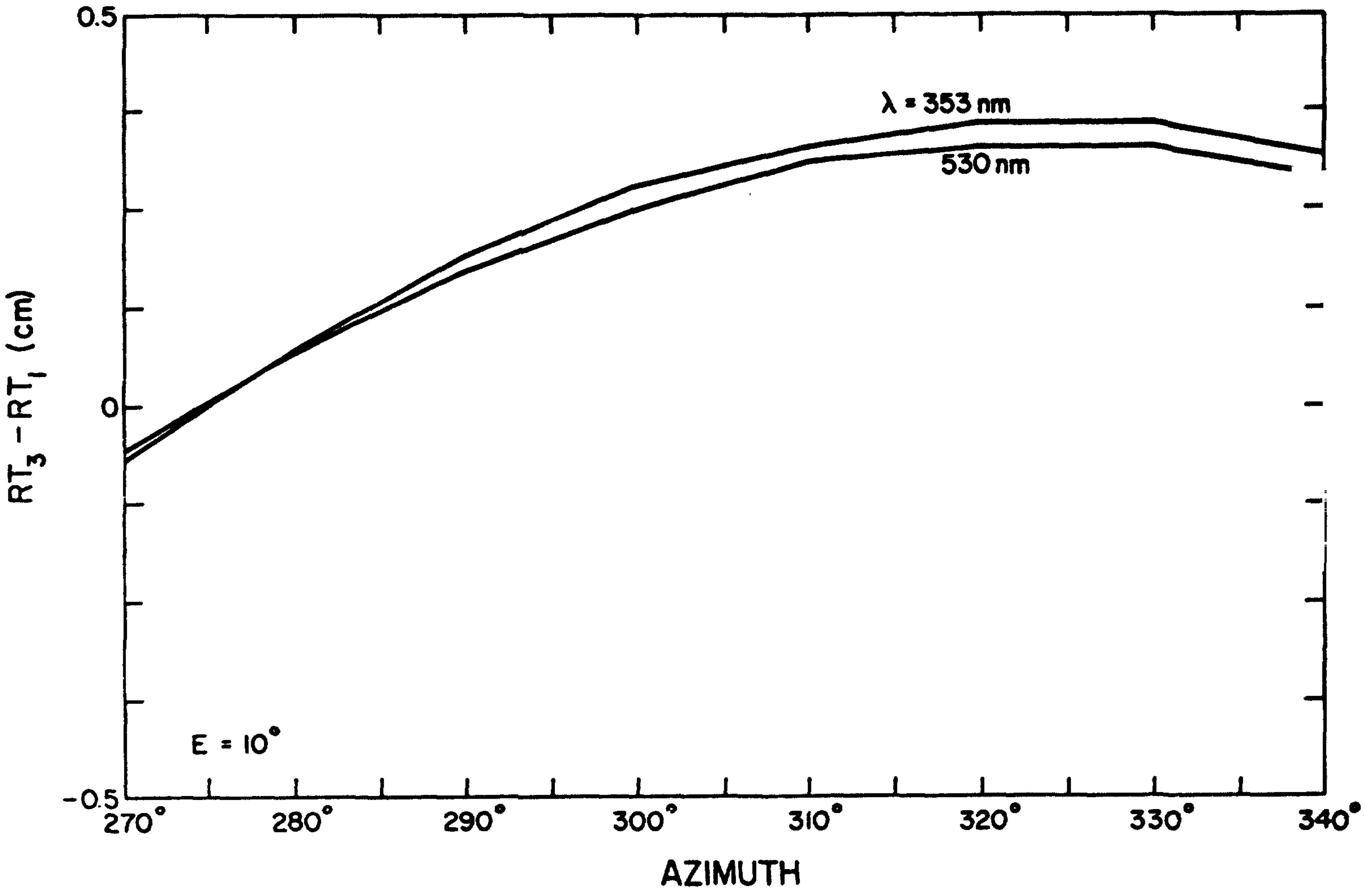

Figure 32. Comparison of RT_3-RT_1 at λ = 353 and 530 nanometers. E = 10°. Laser site = 58. Aux. sites = 52 and 55. Time = 2:30. Date = 1/27/70.

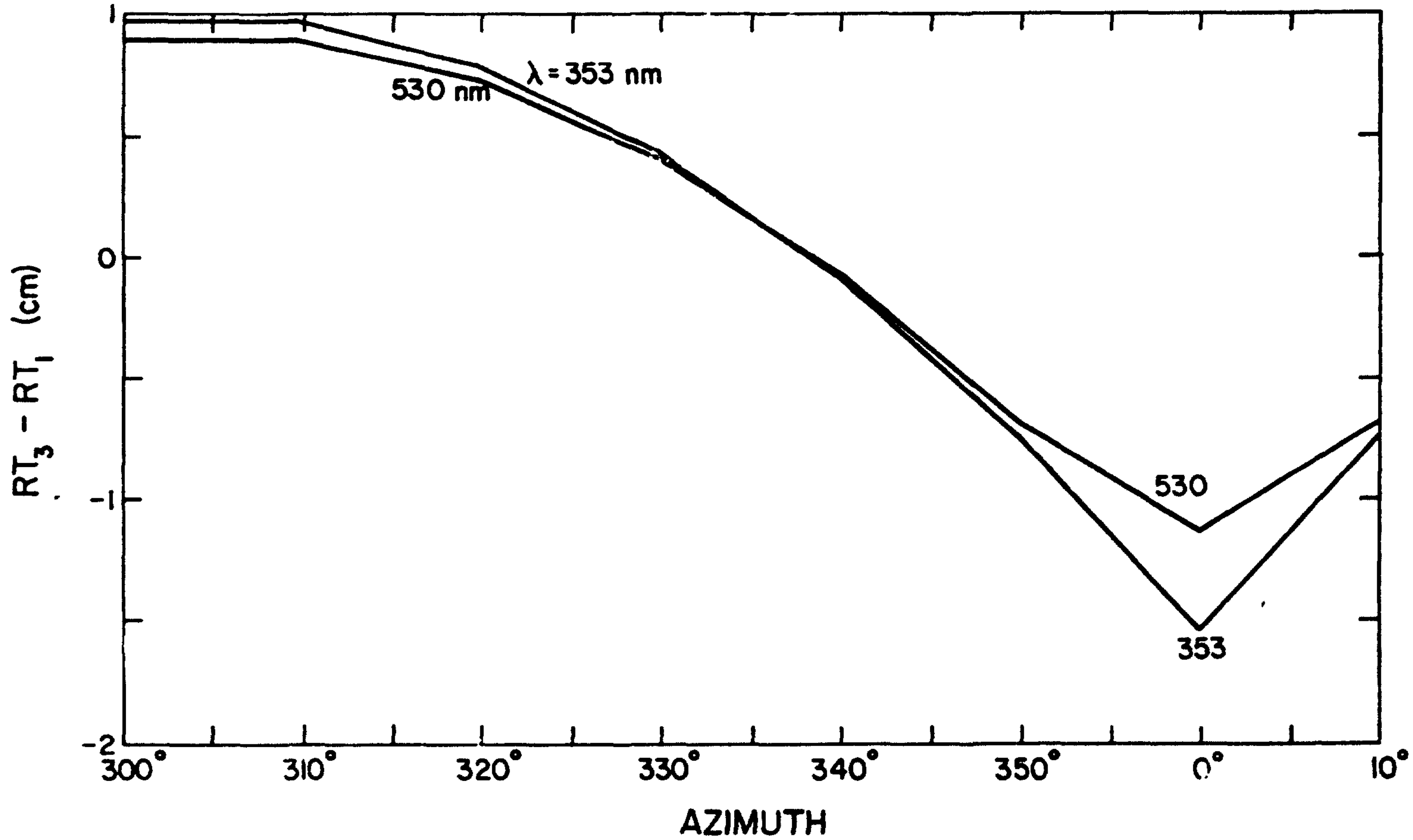

Figure 33. Comparison of RT_3-RT_1 at λ =353 and 530 nanometers.
E =10°. Laser site = 56. Aux. sites = 52 and 55.
Time = 21:30. Date = 1/30/70.

6. ACCURACY OF RADIOSONDE BALLOON DATA

The results in Sections 4 and 5 indicate that the asymmetries in atmospheric refractivity can introduce substantial error into the surface correction formulas. This is true even though the asymmetries appear to be small. The variations in refractivity at a given height measured by the three radiosondes along the balloon trajectories and directly above the release sites are only on the order of 1 to 2 percent (Figures 7, 8, 10, and 11). Measured pressure differences are typically a few millibars and temperature differences are typically two degrees or less. These small differences are not substantially greater than the accuracy of the radiosonde measurements. The question then arises, "Are the effects we observed in Sections 4 and 5 due to actual asymmetries in the atmosphere or are they caused by errors in the radiosonde measurements?" To answer this question we introduced random errors into some selected samples of the radiosonde data and observed the effects on the one-dimensional and three-dimensional ray trace results.

Typical radiosonde measurement errors [5] are summarized in Table 4. These errors were introduced by adding statistically independent random numbers to the radiosonde measurements. The random numbers were Gaussian distributed with a zero mean and standard deviation equal to the root mean square error of the corresponding measurement. The temperature error had a standard deviation of .7 (^{o}C). The pressure error had a standard deviation which was a function of height (Example from Table 4: height = 35,000 feet then the standard deviation = 1 (mb.)). The relative humidity standard deviation was a function of temperature. If the temperature was below -40 ^{o}C a standard deviation of 10 (%) was assumed. The temperature and pressure errors were just added to each measurement, while the relative humidity error was a percentage error. This procedure of calculating the random errors

TABLE 4

STANDARD ACCURACIES OF RADIOSONDE BALLOON MEASUREMENTS [5]

Parameter	Operating Range	Accuracy (Root Mean Square)
Temperature	Surface - 120,000 feet	0.7 °C
Relative Humidity	$T > 0^{\circ}C$	5%
	$0^{\circ} \geq T \geq -40^{\circ}C$	10%
	$T < -40^{\circ}C$	Questionable
Pressure	Surface - 10,000 feet	.7 mb.
	10,000 - 20,000 feet	1.0 mb.
	20,000 - 30,000 feet	1.2 mb.
	30,000 - 40,000 feet	1.0 mb.
	40,000 - 50,000 feet	.7 mb.
	50,000 - 60,000 feet	.55 mb.

was done for each measurement. Each Gaussian error calculated was independent of any other error.

Comparisons were made between ray traces through refractivity profiles generated by error free data (original radiosonde data) and error data (Gaussian errors introduced). The error data consisted of two types, one with pressure only errors, the other with temperature, pressure, and relative humidity errors. Table 5 summarizes the results obtained from these ray traces. $STD(RT_3-RT_1)$ was obtained from ray traces through refractivity profiles generated by error free data, while $STD(RT_3''-RT_1'')$ was calculated from ray traces through refractivity profiles generated by error data (pressure, temperature, and relative humidity errors). The sample size for these standard deviations was 80. Notice that the ratio of $STD(RT_3-RT_1)$ to $STD(RT_3''-RT_1'')$ is 5 at $E = 10^{\circ}$. This ratio can be regarded as the effective

TABLE 5

RESULTS OBTAINED FROM INVESTIGATION OF TYPICAL ERRORS BEING INTRODUCED INTO RADIOSONDE BALLOON DATA

	STANDARD DEVIATION (cm)							
	RT_3-RT_1	$(RT_3-RT_1) - (RT_3''-RT_1'')$	RT_3-RT_3''	RT_3-RT_3'	$MM-RT_1$	$MM''-RT_1''$	RT_1-RT_1''	$MM-MM''$
$E=80^\circ$	0.26	0.19	0.24	0.11	0.08	0.07	0.24	0.08
$E=40^\circ$	0.47	0.23	0.31	0.20	0.11	0.12	0.30	0.12
$E=20^\circ$	1.09	0.33	0.55	0.27	0.26	0.25	0.44	0.23
$E=10^\circ$	3.52	0.72	1.05	0.45	0.63	0.71	0.68	0.50
N	80	80	80	40	12	12	12	12

RT_3'' = 3-dimensional refractivity ray trace with typical errors in pressure, temperature and relative humidity.

RT_3' = 3-dimensional refractivity ray trace with typical errors in pressure.

RT_1'' = Spherically symmetric refractivity ray trace with typical errors in pressure, temperature, and relative humidity.

RT_1' = Spherically symmetric refractivity ray trace with typical errors in pressure.

MM'' = Marini and Murray's formula correction with typical errors in pressure, temperature, and relative humidity.

N = Sample Size

"signal-to-noise ratio" (SNR). This "SNR," though, decreases to 1.38 @ E = 80°. The standard deviations of RT_3-RT_3'' and RT_3-RT_3' were calculated to examine what percentage of the change in ray traces was caused by pressure errors. The indications are that errors in pressure can account for approximately 50% of the total change in ray traces. The standard deviation of MM-RT_1 is approximately equal to STD(MM"-RT_1''). This means Marini and Murray's formula is somewhat self-compensating when random errors are introduced into the radiosonde measurements. It is interesting to compare the standard deviations of RT_1-RT_1'' and RT_3-RT_3''. At an elevation angle of 10° STD(RT_3-RT_3'') is greater than STD(RT_1-RT_1''). This can be accounted for by the fact that for RT_3'' random errors were introduced into three balloon releases which were then used to calculate the refractivity along the ray path, while the refractivity along the ray path for RT_1'' was calculated using only one. At 80° STD(RT_3-RT_3'') is equal to STD(RT_1-RT_1'') because the refractivity is primarily a function of the refractivity measured by the balloon released from the laser site. The standard deviation of MM-MM" represents the magnitude of the error introduced into Marini and Murray's formula by surface measurement errors. It is interesting to note that STD(MM-MM") is approximately equal to STD(MM-RT_1) at each elevation angle. Figures 34 through 39 are graphs of RT_3-RT_1, RT_3'-RT_1'' versus azimuth. The "SNR" is similar between Figures 34 and 35. Notice how RT_3''-RT" varies around RT_3-RT_1 in Figure 39.

At an elevation angle of 10° the change in the ray trace results caused by errors in radiosonde data (STD(RT_3''-RT_1'')) is only 20% of the total range correction error (STD(RT_3-RT_1)). These results indicate that Gaussian errors in radiosonde measurements are not the primary source of the refractivity gradients which were observed in Sections 4 and 5.

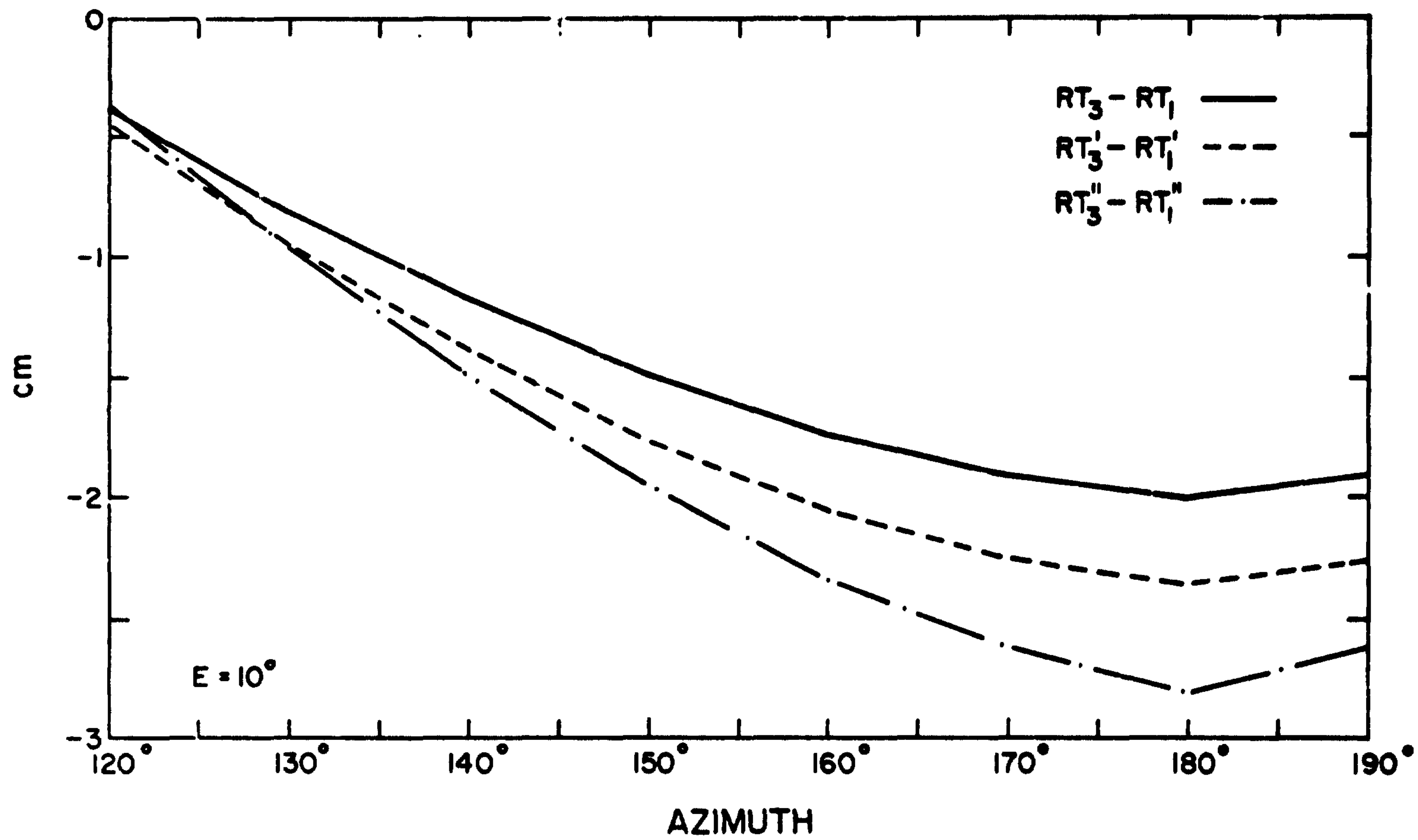

Figure 34. Plots of RT_3-RT_1 without random errors (———), with pressure errors (- - -), and with pressure, temperature, and relative humidity errors (-·-·-). E = 10°. Laser site = 56. Aux. sites = 52 and 54. Time = 23:30. Date = 1/21/70.

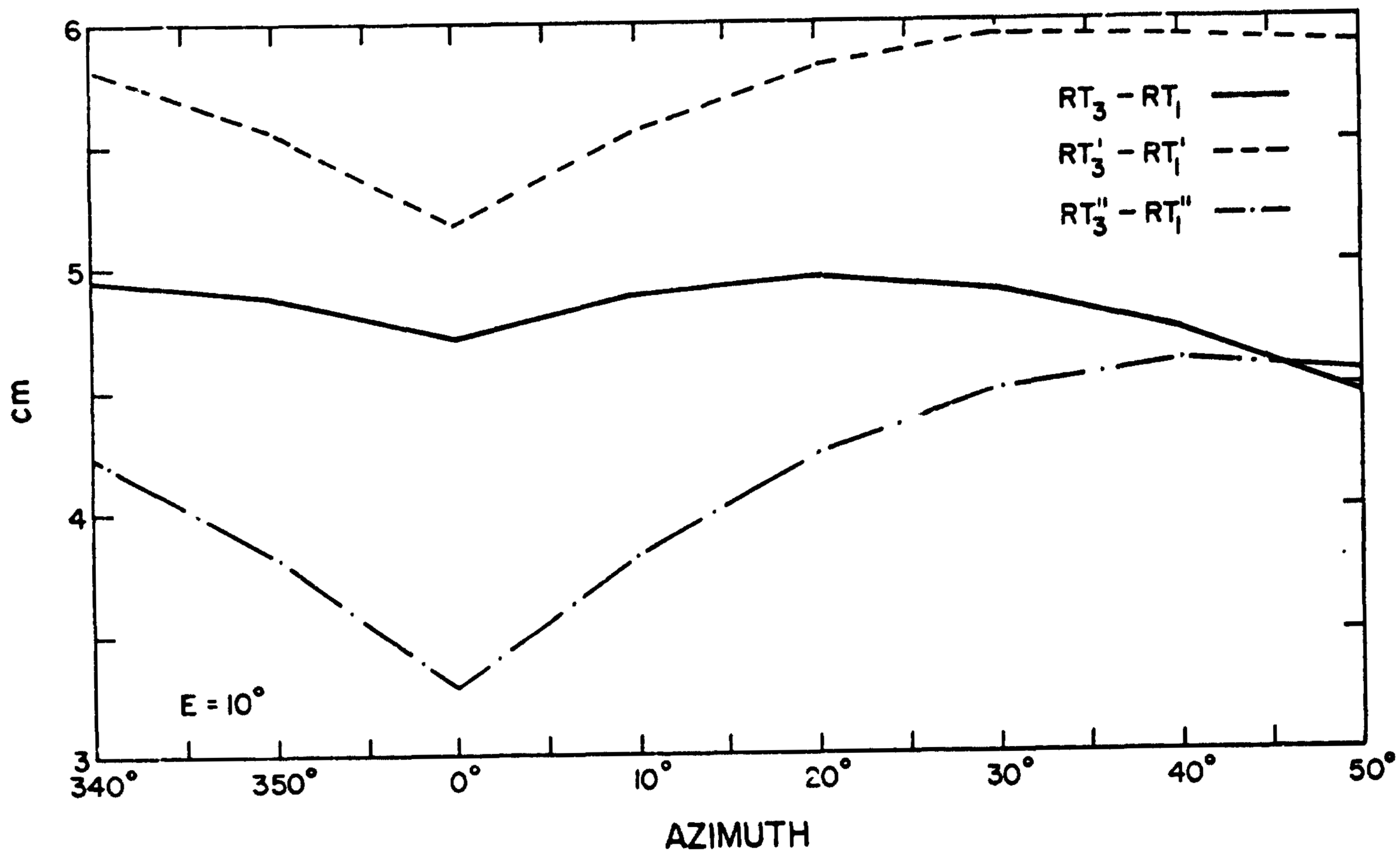

Figure 35. Plots of RT_3-RT_1 without random errors (———), with pressure errors (- - -), and with pressure, temperature, and relative humidity errors (-·-·-). E = 10°. Laser site = 52. Aux. sites = 55 and 56. Time = 21:30. Date = 1/21/70.

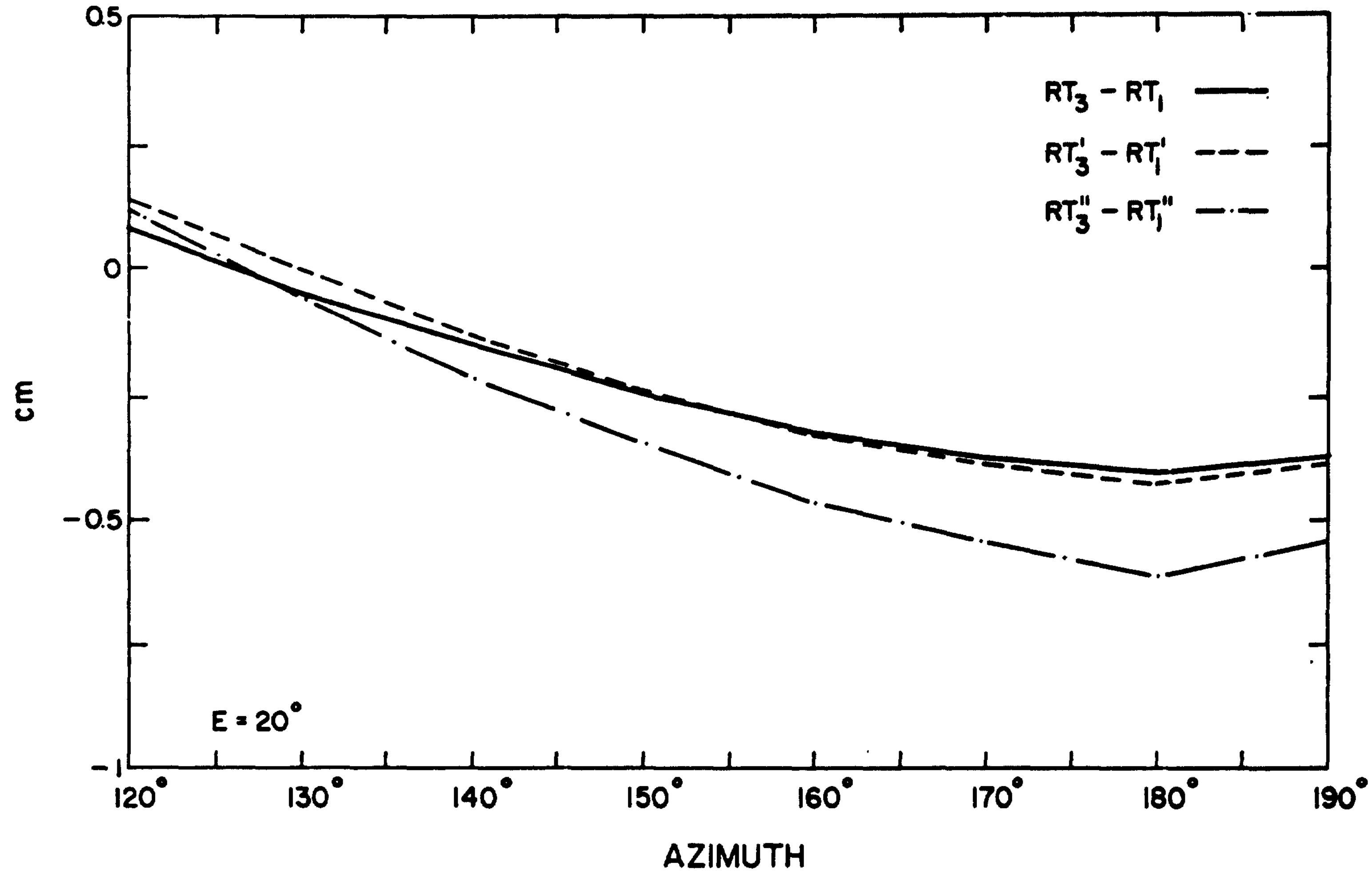

Figure 36. Plots of RT_3-RT_1 without random errors (——), with pressure errors (- - -), and with pressure, temperature, and relative humidity errors (-·-·-). E = 20°. Laser site = 56. Aux. sites = 52 and 54. Time = 23:30. Date = 1/21/70.

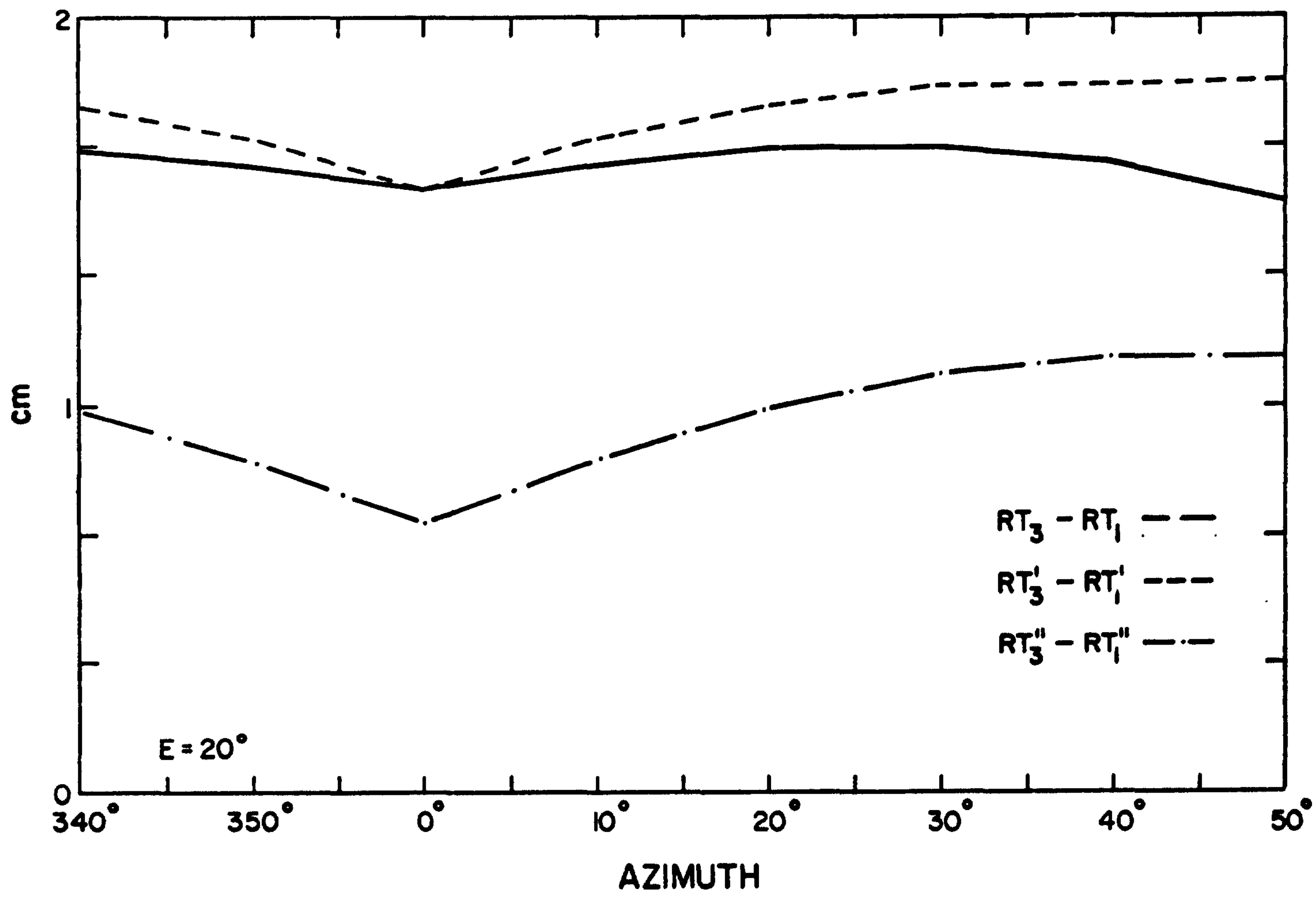

Figure 37. Plots of RT_3-RT_1 without random errors (———), with pressure errors (- - -), and with pressure, temperature, and relative humidity errors (-.-.-). E = 20°. Laser site = 52. Aux. sites = 55 and 56. Time = 23:30. Date = 1/21/70.

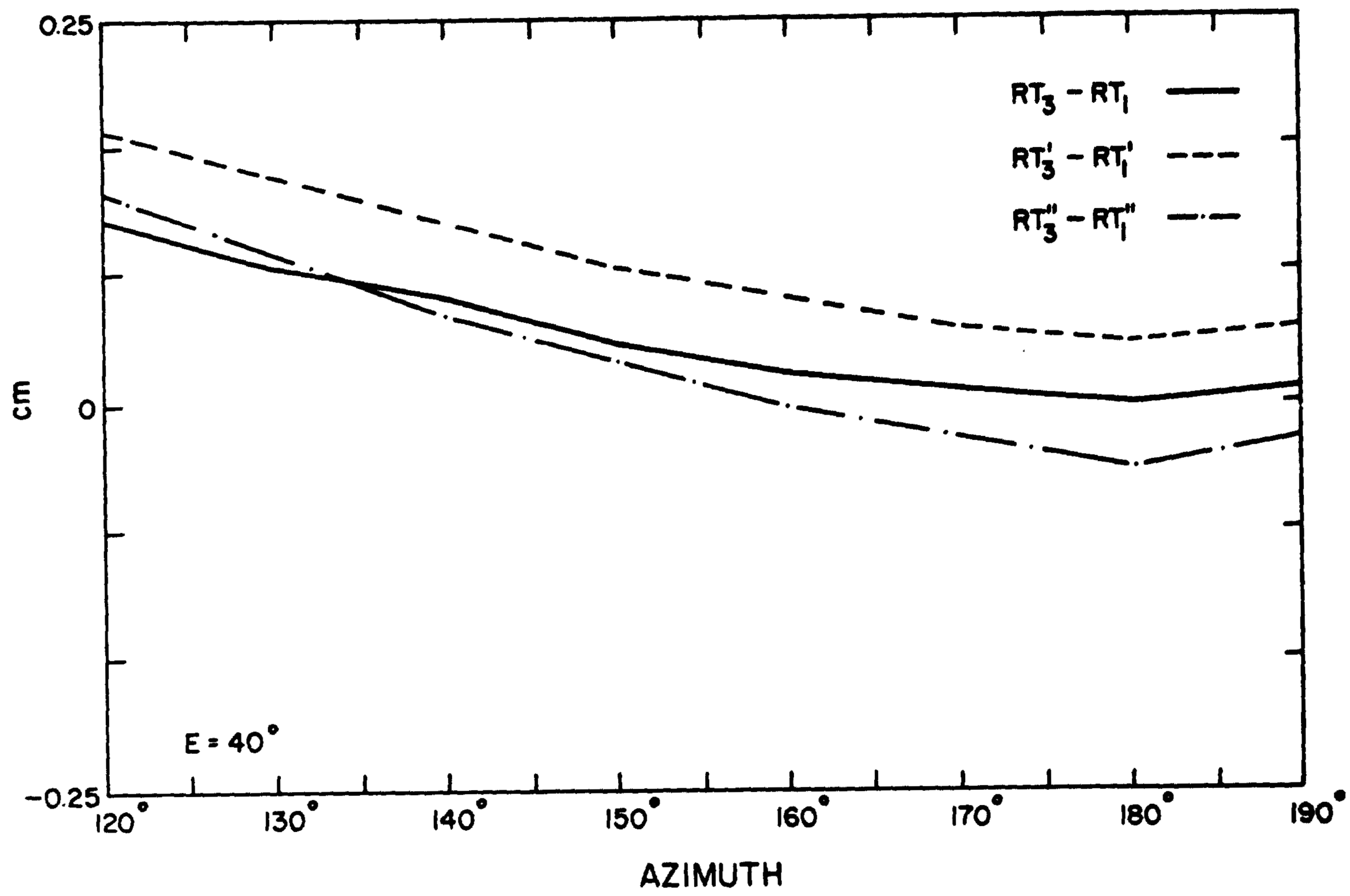

Figure 38. Plots of RT_3-RT_1 without random errors (———), with pressure errors (- - -), and with pressure, temperature, and relative humidity errors (-·-·-). E = 40°. Laser site = 52. Aux. sites = 55 and 56. Time = 23:30. Date = 1/21/70.

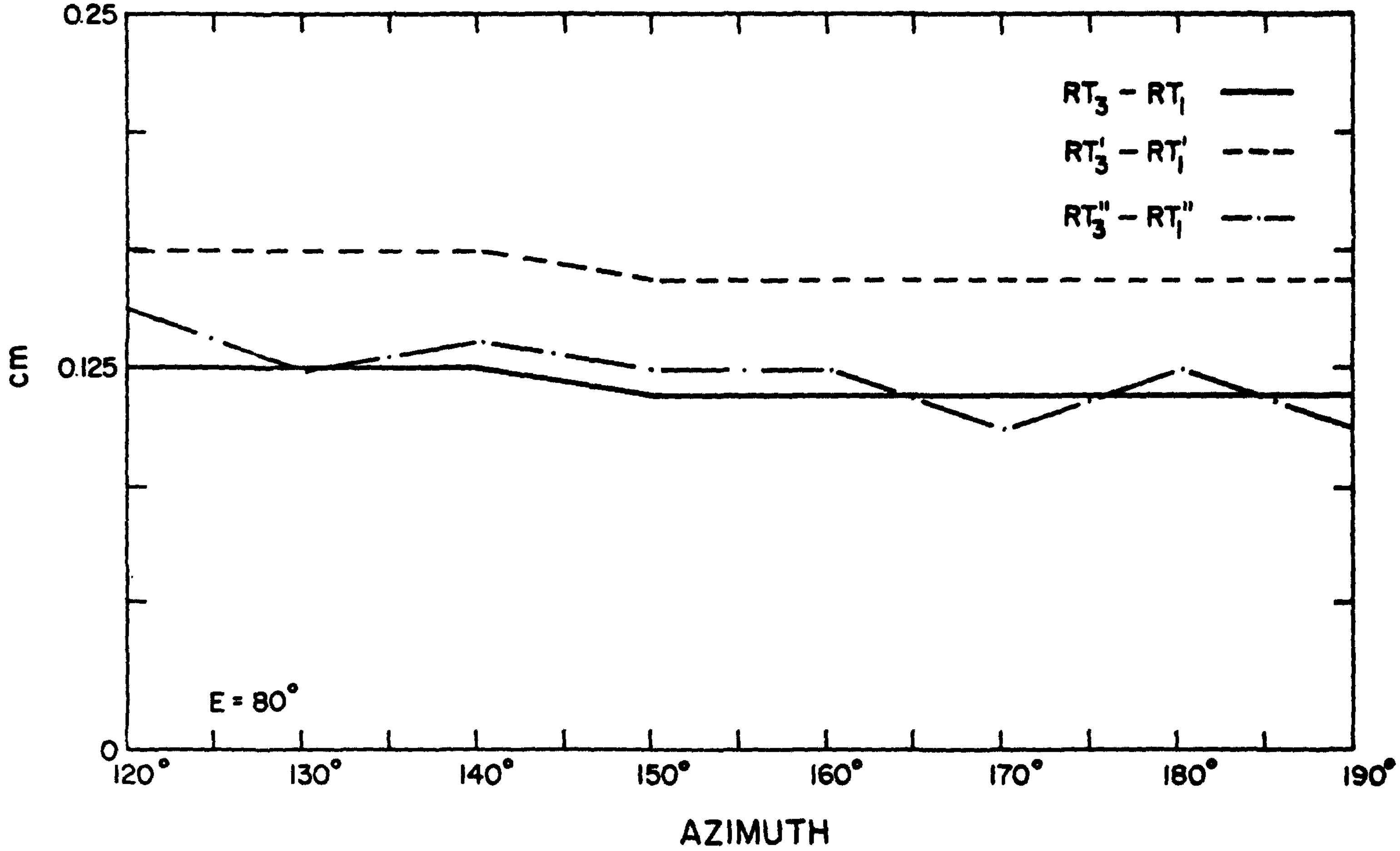

Figure 39. Plots of RT_3-RT_1 without random errors (———), with pressure errors (- - -), and with pressure, temperature, and relative humidity errors (-·--·-). E = 80°. Laser site = 52. Aux. sites = 55 and 56. Time = 23:30. Date = 1/21/70.

7. SURFACE CORRECTION FORMULA

In this section a surface correction formula will be developed to partially compensate for the asymmetries in the refractivity. Using Figure 40, formula (7-1) can be obtained for the refractivity at any value of h and α. Figure 40 is on page 57. This formula for group refractivity assumes refractivity varies linearly in the horizontal direction

$$N_g(h) = N_{g_1}(h)\left(\frac{\psi - \alpha}{\psi}\right) + N_{g_2}(h)\left(\frac{\alpha}{\psi}\right) \qquad (7\text{-}1)$$

where ψ is the angular separation between the two sites, and N_{g_1} (h) and N_{g_2} (h) are the group refractivities directly above their respective sites. Using Figure 41 and approximating β by E and γ by 90°, the chord distance between the two surface points can be calculated. Assuming the chord distance

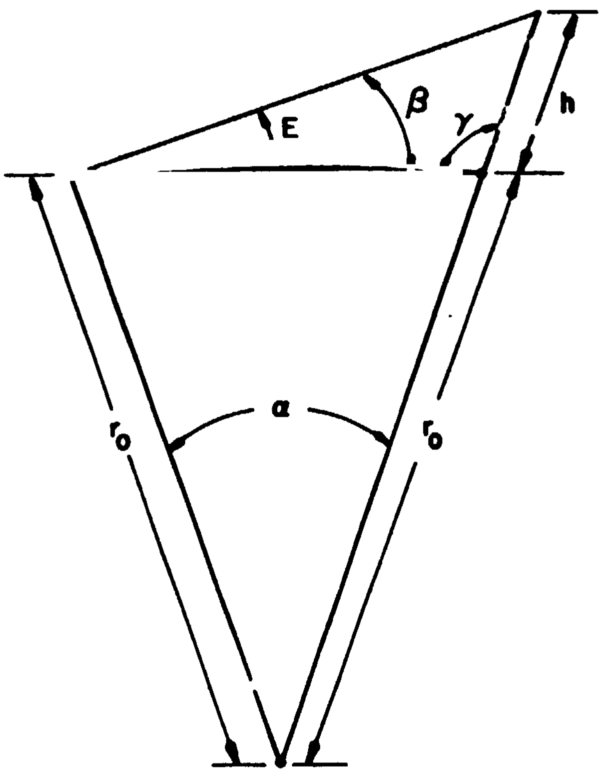

Figure 41. Geometry for approximations made.

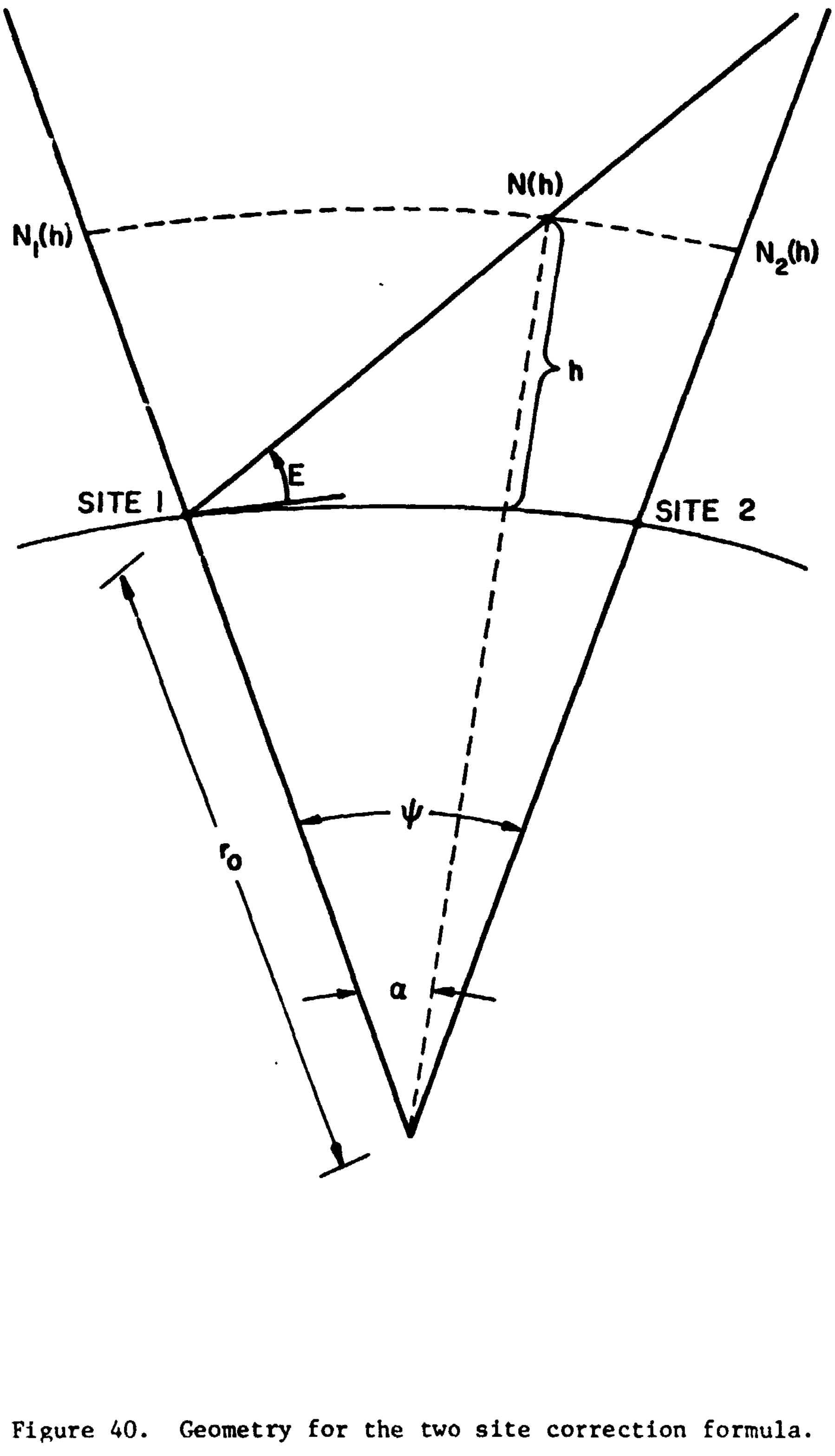

Figure 40. Geometry for the two site correction formula.

is equivalent to the arc distance, α can be approximated by $h/(r_0 \cdot \tan E)$. Applying this approximation and rearranging terms simplifies (7-1) to

$$N_g(h) = N_{g_1}(h) + \left[\frac{N_{g_2}(h) - N_{g_1}(h)}{\psi}\right] \cdot \left[\frac{h}{r_0 \tan E}\right] \tag{7-2}$$

where

r_0 = nominal earth radius = 6378 km .

Equation (7-2) for group refractivity can now be substituted into integral (2-8) to obtain

$$\Delta R = 10^{-6} \int_{r_0}^{r_1} \frac{r}{r_0 \tan E}\left[\frac{N_{g_2}(r) - N_{g_1}(r)}{\psi \sin \theta}\right] dr + 10^{-6} \int_{r_0}^{r_1} \frac{N_{g_1}(r)}{\sin \theta}\, dr$$

$$+ \left[\int_{r_0}^{r_1} \frac{dr}{\sin \theta} - R\right] \quad . \tag{7-3}$$

Notice that the last two terms are just the integrals which Marini and Murray evaluated to obtain their correction formula for site 1, MM_1. The first term is an error term which compensates for linear variations in refractivity in the horizontal direction. Thus we can write

$$\Delta R \approx \frac{10^{-6}}{r_0 \psi \tan E}\left[\int_{r_0}^{r_1} \frac{N_{g_2}(r)}{\sin \theta}\, r\, dr - \int_{r_0}^{r_1} \frac{N_{g_1}(r)}{\sin \theta}\, r\, dr\right] + MM_1 \quad . \tag{7-4}$$

Using Snell's Law [3] and expanding in inverse powers of $\sin(\theta_0)$ gives

$$10^{-6} \int \frac{N_g(r)}{\sin \theta}\, r\, dr \doteq \frac{10^{-6}}{\sin \theta_0} \int hN_g(h)\, dh - \frac{1}{\sin^3 \theta_0}\left\{\frac{10^{-6}}{r_0} \int h^2 N_g(h)\, dh\right.$$

$$\left. - 10^{-12} \int hN_g(h)[N_0 - N(h)]\, dh\right\} \tag{7-5}$$

where

$h = r - r_0$.

N is the phase refractivity, and is given at optical frequencies [3] by

$$N = 77.6 \frac{P}{T} \left[1 + \frac{7.52 \times 10^{-3}}{\lambda^2} \right] . \tag{7-6}$$

An approximate evaluation of the first term of the expansion in equation (7-5) was calculated by Marini and Murray [3]

$$10^{-6} \int hN_g(h)\, dh = f(\lambda)\, r_0 (1.084 \times 10^{-8})\, P_0 T_0 K \tag{7-7}$$

where F_0, T_0, and K are as defined in an earlier section. The remaining terms of the expansion will be neglected since their contribution is only a small percentage of the final correction (millimeters compared to centimeters). Combining Equations (7-4), (7-5) and (7-7) gives

$$\Delta R = MM_1 + \frac{f(\lambda)(1.084 \times 10^{-8})}{\psi \tan E \sin E} \quad (P_2 T_2 K_2 - P_1 T_1 K_1) \tag{7-8}$$

where MM_1 is the Marini and Murray correction for site 1 (equation (2-9)) and $P_1T_1K_1$ and $P_2T_2K_2$ are a function of the surface measurements at sites 1 and 2 respectively. These measurements are taken at identical heights above sea level. The fact that θ_o can be approximated by E was also used in equation (78). Noting that $\psi = d/r_0$, where d is the arc distance between the two sites, Equation (7-8) becomes

$$\Delta R = MM_1 + \frac{C}{\tan E \sin E} \tag{7-9}$$

where

$$C = \frac{f(\lambda)(1.084 \times 10^{-8})\, r_0}{d} \; (P_2 T_2 K_2 - P_1 T_1 K_1) \quad .$$

Formula (7-9) was used to correct a small number ($N = 10$) of laser ranging measurements. This was done by selecting the azimuth angle for RT_3 so that the laser beam propagated directly over one of the other release sites. The first two radiosonde data points of this site were used to interpolate the pressure, temperature, and relative humidity at the same height as the laser site. In all cases the height of the laser site was greater than the remaining two release sites. These values were used to calculate $P_2T_2K_2$ in formula (7-8). $P_1T_1K_1$ was calculated from the first radiosonde data point of the balloon released from the laser site. The standard deviation of the difference between Marini and Murray's formula and RT_3 was 2.43 cm and the mean was .31 cm for the 10 ray traces. When equation (7-9) was used the standard deviation was reduced to .69 cm and the mean was .27 cm. An elevation angle of 10° was used because the error in Marini and Murray's formula is largest at this angle. Figure 42 is a graph of $C/(\sin E \cdot \tan E)$ with the circles denoting the values obtained at different elevation angles by ray tracing through a three-dimensional refractivity profile. The constant C was calculated so that at $E = 10^\circ$ the curve $C/(\sin E \cdot \tan E)$ matched the first data point. C was not calculated from equation (7-9) because a second set of surface measurements at a point beneath the laser beam trajectory was not available. The close match between the theoretical curve and the ray trace data points indicates that the error closely follows the $1/(\sin E \cdot \tan E)$ dependence predicted by the theory.

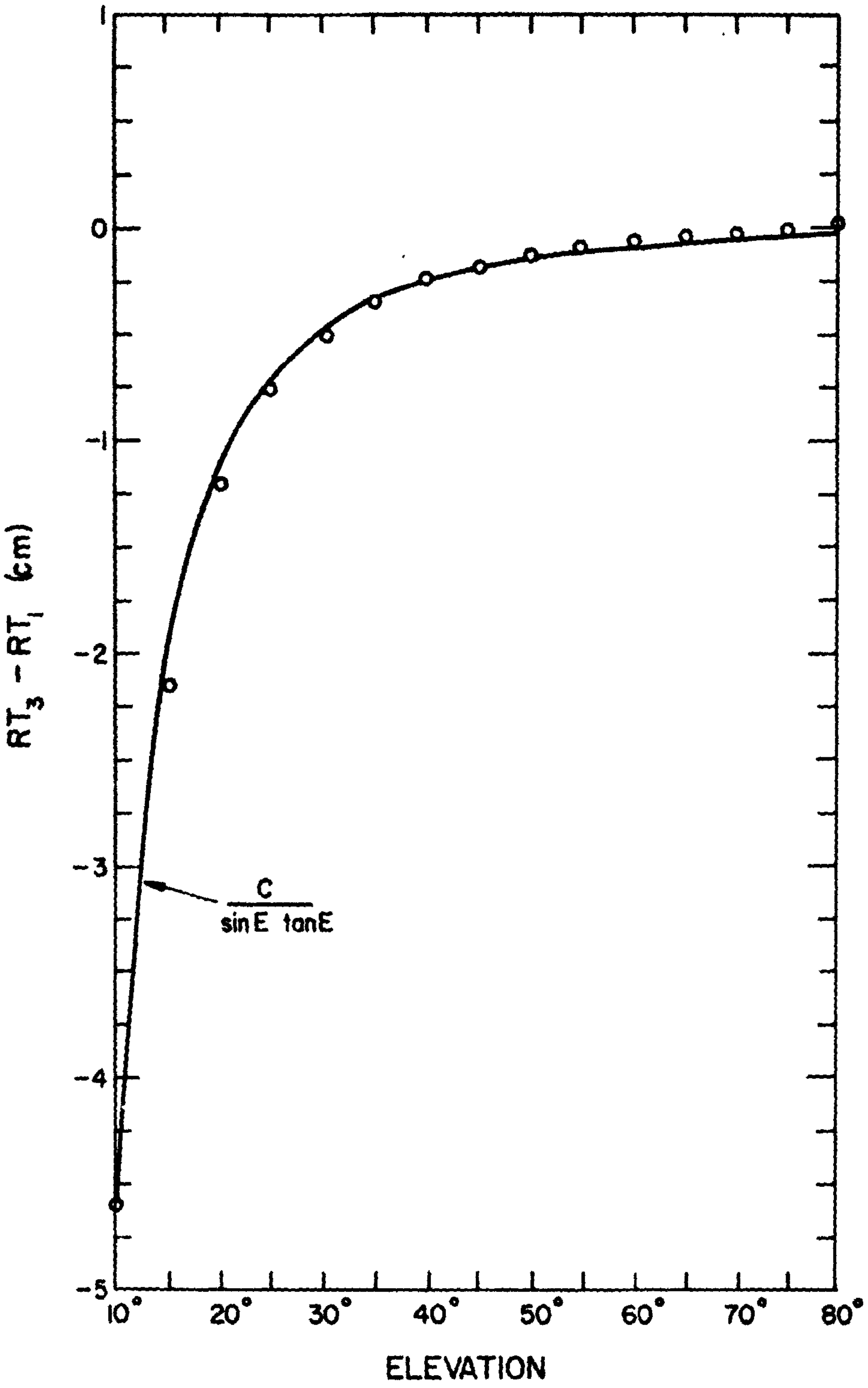

Figure 42. Theoretical dependence (———) of RT_3-RT_1 on elevation angle and actual data points (o) obtained by ray tracing. Azimuth = 140°. Laser site = 52. Aux. sites = 53 and 54. Time = 1:30. Date = 1/20/70.

8. CONCLUSIONS

Numerous correction formulas have been developed to compensate laser ranging data for atmospheric refraction. The formulas developed by Saastamoinen (1) and Marini (2) are particularly convenient for use in correcting satellite tracking data because they use surface measurements of pressure, temperature and humidity to derive the range correction. The formulas were derived by assuming that the refractive index profile in the atmosphere is spherically symmetric.

We have investigated the accuracy of Marini's formula by comparing it with corrections obtained by ray-tracing through 3-dimensional refractivity profiles. The 3-dimensional profiles were generated using measurements obtained from radiosonde balloons which were released simultaneously from three separate locations around the ranging site. The refractivity was assumed to vary linearly in the horizontal direction. Also, the ray tracing method was based in part on the hydrostatic equation because the heights which appear in the radiosonde profiles were calculated from the measured pressures, temperatures and relative humidities using the hydrostatic equation. Our results indicate that the departure of the refractivity profiles from spherical symmetry introduces approximately 3 centimeters error into Marini's correction formula at 10° elevation. The error decreases to a few millimeters near zenith. For a given 3-dimensional profile, the error is a function of azimuth and elevation angles and can vary over a range of more than 5 centimeters for a 90° change in azimuth. The error decreases as the wavelength of the laser is increased.

A surface correction formula which compensates for horizontal variations in refractivity has been developed. The formula requires the pressure, temperature and relative humidity at the laser site and at a second location beneath the laser beam trajectory. Although the formula has been tested on only a small sample of data, the results are encouraging.

APPENDIX A

RAY TRACING PROGRAM

The first step in the ray tracing program is to read the three sets of radiosonde measurements. The measurements are used to generate three tables of pressure, temperature and relative humidity at fixed altitudes along the balloon trajectories. Since the altitudes at which the radiosonde measurements were obtained will in general be different from the fixed altitudes and different for each balloon release, the pressure, temperature and relative humidity at the fixed altitudes must be interpolated. The interpolation procedure is discussed in detail in Section 3. Next the phase and group refractivity at the fixed altitudes along each balloon trajectory are calculated. The phase and group refractivities along the ray path are then calculated using the interpolating formula (3-3) and equation (3-4). The co-latitude and longitude of the ray at the fixed altitude are determined by assuming the ray path is a straight line. At 10^{o} elevation and above, the error introduced by this assumption is negligible. The group and phase refractivities at the fixed altitudes along the ray path are passed to the ray tracing program which calculates range (RT_3), range correction and θ_o (see Figure 1). The value of θ_o is used in Marini and Murray's formula. RT_1 is calculated using only the measurements obtained by the balloon which was released from the laser site. The Thayer method ray tracing program which was used by Marini and Murray [3] was also used to obtain our results. Therefore, our procedure for calculating RT_1 is the same as Marini and Murray's. Initially, a more complex ray tracing program which required the θ and ϕ refractivity gradients was used to calculate the range correction. However, the results were not substantially different from those obtained from the simpler and faster Thayer method program. A flowchart of the ray-tracing procedure is illustrated in Figure 43.

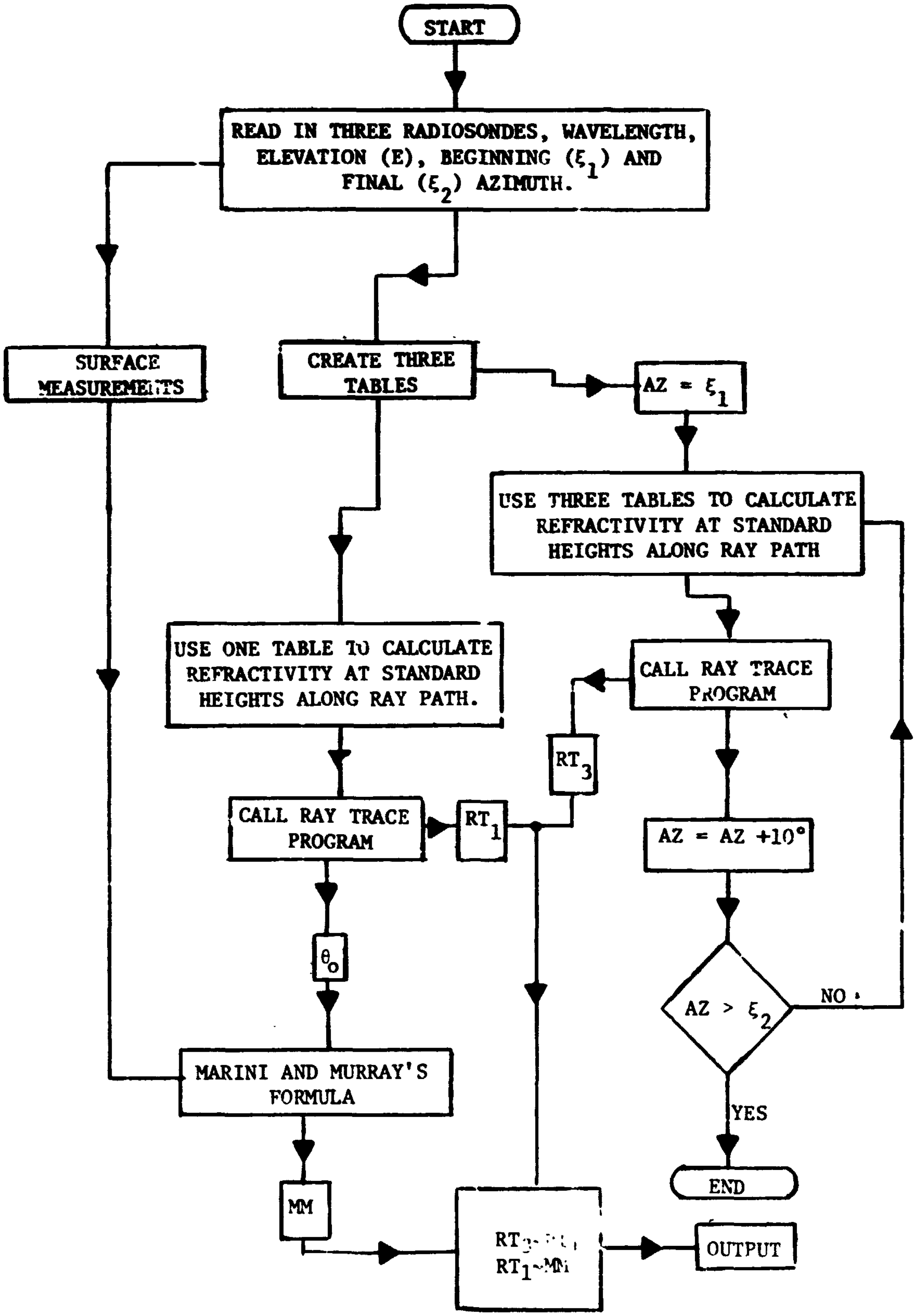

Figure 43. Flow chart of ray tracing procedure.

APPENDIX B

RAY TRACE RESULTS

The results of the ray traces through the three-dimensional and spherically symmetric refractivity profiles are listed on pages 67 through 144. Sheet 1 of the data summary lists the radiosonde parameters for each set of three balloon releases. The parameters include the release date and time and the altitude, azimuth angle, elevation angle and range of the balloon when the last measurement was taken. The ray trace parameters are listed on sheets 2, 3 and 4. RT_3-RT_1 is the difference between the three-dimensional and spherically symmetric refractivity ray traces. MM in the correction obtained from Marini and Murray's formula, eq. (2-9). MM-RT_1 is the difference between Marini and Murray's formula and the ray trace through the spherically symmetric profile. Azimuth selections for RT_3 are from the beginning azimuth ξ_1 to the final azimuth ξ_2 in 10° increments. Table 6 is used to calculate ξ_1 and ξ_2 as explained in Section 5. The arc distance between release sites can be obtained from Table 7.

TABLE 6

AZIMUTH ANGLES BETWEEN RELEASE SITES

Release Sites	Release Sites							
	51	52	53	54	55	56	57	58
51	xx	239°	133°	193°	228°	205°	178°	161°
52	31°	xx	85°	135°	223°	177°	156°	130°
53	313°	265°	xx	229°	243°	224°	197°	179°
54	13°	315°	49°	xx	259°	215°	172°	114°
55	48°	43°	63°	79°	xx	135°	126°	95°
56	35°	357°	44°	35°	315°	xx	118°	75°
57	2°	346°	17°	352°	306°	298°	xx	41°
58	314°	310°	359°	294°	275°	255°	229°	xx

TABLE 7

ARC DISTANCE BETWEEN RELEASE SITES (KILOMETERS)

Release Sites	Release Sites							
	51	52	53	54	55	56	57	58
51	xx	130.49	79.97	159.94	242.02	256.76	284.95	202.03
52	130.49	xx	181.00	101.03	119.95	189.40	248.33	220.98
53	79.97	181.00	xx	138.07	242.02	256.76	258.85	143.10
54	159.94	101.03	138.07	xx	145.22	119.95	151.53	159.11
55	242.02	119.95	242.02	145.22	xx	99.75	199.94	248.33
56	256.76	189.40	256.76	119.95	99.75	xx	105.23	180.57
57	284.95	[illegible].33	258.85	151.53	199.94	105.23	xx	130.49
58	202.03	220.98	143.10	159.11	248.33	180.57	13[illegible].49	xx

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/30/70

	STATION	STATION	STATION
#	52	55	56
RELEASE TIME	21:30	21:30	21:30
ALTITUDE (km)	15.9	16.1	13.1
AZIMUTH	76.7	82.4	73.0
ELEVATION	7.5	7.1	6.5
RANGE (km)	121.4	129.4	118.4

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 1/30/70 TIME 21:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10°	20°	40°	80°
RT_1-RT_3 (cm)	150	-4.23	-2.20	-1.17	-.76
	160	-5.16	-2.47	-1.23	-.76
	170	-6.04	-2.73	-1.30	-.77
	180	-6.86	-2.97	-1.36	-.77
	190	-6.04	-2.73	-1.30	-.77
	200	-5.16	-2.47	-1.23	-.76
	210	-4.23	-2.20	-1.18	-.76
	220	-3.28	-1.93	-1.10	-.75
SITE STN # 52	MM (m)	13.59	7.04	3.77	2.46
	MM-RT_1 (cm)	0.43	0.39	0.18	0.12

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 55 RELEASE DATE 1/30/70 TIME 21:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	40	1.26	-.66	-.82	-.75
	50	1.44	-.60	-.80	-.75
	60	1.45	-.60	-.80	-.74
	70	1.31	-.64	-.81	-.75
	80	1.00	-.73	-.83	-.75
	90	0.54	-.86	-.86	-.75
	100	-.06	-1.03	-.90	-.76
	110	-.78	-1.24	-.95	-.76
SITE STN # 55	MM (m)	13.57	7.04	3.76	2.46
	MM-RT_1 (cm)	0.20	0.35	0.17	0.12

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 56 RELEASE DATE 1/30/70 TIME 21:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	300	0.90	-.90	-.97	-.84
	310	0.89	-.90	-.97	-.84
	320	0.73	-.95	-.98	-.84
	330	0.40	-1.05	-1.00	-.84
	340	-.08	-1.19	-1.04	-.85
	350	-.69	-1.37	-1.08	-.85
	360	-1.14	-1.59	-1.13	-.85
	370	-.69	-1.37	-1.08	-.85
SITE STN # 56	MM (m)	13.56	7.03	3.76	2.46
	MM-RT_1 (cm)	0.78	0.57	0.27	0.19

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/27/70

	STATION	STATION	STATION
#	52	55	58
RELEASE TIME	2:30	2:30	2:30
ALTITUDE (km)	15.4	15.9	16.0
AZIMUTH	86.6	93.5	84.[illegible]
ELEVATION	7.8	7.0	10.3
RANGE (km)	113.6	130.6	90.0

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 1/27/70 TIME 2:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	130	-.88	-.44	-.24	-.17
	140	-1.05	-.49	-.25	-.17
	150	-1.21	-.53	-.26	-.17
	160	-1.35	-.57	-.27	-.18
	170	-1.48	-.61	-.28	-.18
	180	-1.60	-.64	-.28	-.18
	190	-1.48	-.61	-.28	-.18
	200	-1.35	-.57	-.27	-.18
SITE STN # 52	MM (m)	13.53	7.02	3.75	2.45
	$MM-RT_1$ (cm)	-.10	0.13	0.05	0.04

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 55 RELEASE DATE 1/27/70 TIME 2:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10°	20°	40°	80°
RT_1-RT_3 (cm)	20	0.29	-.10	-.13	-.10
	30	0.33	-.09	-.12	-.10
	40	0.33	-.09	-.12	-.10
	50	0.31	-.09	-.12	-.10
	60	0.26	-.10	-.13	-.10
	70	0.18	-.12	-.13	-.10
	80	0.07	-.14	-.14	-.10
	90	-.05	-.18	-.14	-.10
SITE STN # 55	MM (m)	13.51	7.01	3.75	2.45
	MM-RT_1 (cm)	-.11	0.17	0.07	0.04

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 58 RELEASE DATE 1/27/70 TIME 2:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	270	-.06	-.18	-.15	-.13
	280	0.07	-.15	-.14	-.13
	290	0.17	-.12	-.13	-.13
	300	0.25	-.10	-.13	-.13
	310	0.31	-.09	-.13	-.13
	320	0.33	-.09	-.13	-.13
	330	0.33	-.09	-.13	-.13
	340	0.29	-.10	-.13	-.13
SITE STN # 58	MM (m)	13.50	7.00	3.75	2.45
	MM-RT_1 (cm)	-.05	0.16	0.06	0.05

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/21/70

	STATION	STATION	STATION
#	52	54	56
RELEASE TIME	21:32	21:47	21:34
ALTITUDE (km)	15.9	13.2	12.6
AZIMUTH	106.1	104.3	96.9
ELEVATION	7.2	6.4	5.1
RANGE (km)	127.1	119.1	142.1

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 1/21/70 TIME 21:32

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	120	-.39	0.08	0.12	0.13
	130	-.81	-.05	0.09	0.13
	140	-1.17	-.15	0.07	0.13
	150	-1.49	-.25	0.04	0.12
	160	-1.74	-.33	0.02	0.12
	170	-1.91	-.38	0.01	0.12
	180	-2.01	-.41	0.00	0.12
	190	-1.91	-.38	0.01	0.12
SITE STN # 52	MM (m)	13.78	7.14	3.82	2.49
	MM-RT_1 (cm)	0.76	0.46	0.20	0.12

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 54 RELEASE DATE 1/21/70 TIME 21:47

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10°	20°	40°	80°
$RT_1 - RT_3$ (cm)	230	-.71	0.00	0.12	0.15
	240	-.30	0.12	0.15	0.15
	250	0.15	0.25	[illegible]	[illegible]16
	260	0.62	0.39	0.[illegible]	[illegible]
	270	1.09	0.52	0.24	[illegible]
	280	1.56	0.65	0.28	0.17
	290	2.00	0.78	0.31	0.17
	300	2.40	0.89	0.33	0.17
	310	2.76	0.99	0.36	0.18
SITE STN # 54	MM (m)	13.76	7.13	3.81	2.49
	$MM-RT_1$ (cm)	1.29	0.68	0.31	0.19

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 56 RELEASE DATE 1/21/70 TIME 21:34

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10°	20°	40°	80°
	340	3.09	0.97	0.29	0.08
	350	3.17	0.98	0.29	0.08
	360	3.15	0.97	0.29	0.08
	370	3.17	0.98	0.29	0.08
	380	3.09	0.97	0.29	0.08
RT_1-RT_3 (cm)	390	2.94	0.93	0.28	0.08
	400	2.72	0.88	0.27	0.08
	410	2.42	0.80	0.25	0.07
SITE STN	MM (m)	13.76	7.13	3.81	2.49
56	MM-RT_1 (cm)	1.31	0.73	0.34	0.23

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/21/70

	STATION	STATION	STATION
#	52	54	56
RELEASE TIME	23:36	23:35	23:31
ALTITUDE (km)	15.9	12.9	13.2
AZIMUTH	107.4	105.4	99.3
ELEVATION	7.7	4.9	6.3
RANGE (km)	118.2	149.4	120.4

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 1/21/70 TIME 23:36

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10°	20°	40°	60°
RT_1-RT_3 (cm)	120	0.65	0.53	0.30	0.21
	130	0.25	0.34	0.26	0.21
	140	-.56	0.16	0.22	0.21
	150	-1.10	0.00	0.17	0.20
	160	-1.56	-.14	0.14	0.20
	170	-1.94	-.27	0.11	0.20
	180	-2.22	-.36	0.08	0.20
	190	-1.94	-.27	0.11	0.20
SITE STN # 52	MM (m)	13 80	7.14	3.82	2.50
	$MM-RT_1$ (cm)	0.94	0.49	0.21	0.13

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 54 RELEASE DATE 1/21/70 TIME 23:35

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	230	0.64	0.65	0.43	0.29
	240	1.28	0.84	0.47	0.30
	250	1.93	1.03	0.52	0.30
	260	2.58	1.22	0.56	0.31
	270	3.20	1.39	0.60	0.31
	280	3.79	1.56	0.64	0.31
	290	4.32	1.70	0.68	0.32
	300	4.77	1.82	0.71	0.32
	310	5.14	1.91	0.73	0.32
SITE STN # 54	MM (m)	13.78	7.13	3.82	2.49
	MM-RT_1 (cm)	1.51	0.72	0.32	0.22

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 56 RELEASE DATE 1/21/70 TIME 23:31

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	340	4.95	1.66	0.56	0.23
	350	4.88	1.62	0.55	0.23
	360	4.71	1.56	0.53	0.22
	370	4.88	1.62	0.55	0.23
	380	4.95	1.66	0.56	0.23
	390	4.89	1.66	0.56	0.23
	400	4.73	1.62	0.55	0.23
	410	4.46	1.52	0.53	0.23
SITE STN # 56	MM (m)	13.79	7.14	3.82	2.50
	MM-RT_1 (cm)	1.46	0.64	0.27	0.17

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 2/13/70

	STATION	STATION	STATION
#	52	56	53
RELEASE TIME	19:30	19:30	19:30
ALTITUDE (km)	15.1	13.2	15.8
AZIMUTH	87.6	83.5	85.6
ELEVATION	6.6	6.2	5.4
RANGE (km)	131.4	122.2	167.9

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 2/13/70 TIME 19:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	90	0.25	-.23	-.26	-.22
	100	-.38	-.41	-.30	-.23
	110	-1.04	-.59	-.34	-.23
	120	-1.70	-.77	-.38	-.23
	130	-2.35	-.95	-.43	-.24
	140	-2.96	-1.12	-.47	-.24
	150	-3.52	-1.28	-.51	-.25
	160	-4.00	-1.41	-.54	-.25
	170	-4.41	-1.53	-.57	-.25
SITE STN # 52	MM (m)	13.67	7.08	3.79	2.48
	MM-RT_1 (cm)	0.57	0.40	0.18	0.12

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 53 RELEASE DATE 2/13/70 TIME 19:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	200	-4.56	-1.71	-.70	-.35
	210	-4.07	-1.57	-.67	-.35
	220	-3.52	-1.42	-.63	-.35
	230	-2.90	-1.24	-.59	-.34
	240	-2.25	-1.06	-.54	-.34
	250	-1.58	-.88	-.50	-.33
	260	-.92	-.70	-.46	-.33
	270	-.28	-.52	-.42	-.32
	280	0.31	-.36	-.38	-.32
SITE STN # 53	MM (m)	13.63	7.07	3.78	2.47
	MM-RT_1 (cm)	0.52	0.36	0.16	0.09

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 56 RELEASE DATE 2/13/70 TIME 19:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	340	2.43	0.28	-.18	-.25
	350	2.35	0.25	-.18	-.25
	0	2.16	0.19	-.20	-.26
	10	2.35	0.25	-.18	-.25
	20	2.43	0.28	-.17	-.25
	30	2.39	0.27	-.18	-.25
	40	2.24	0.24	-.18	-.26
	50	1.98	0.17	-.20	-.26
	60	1.61	0.07	-.22	-.26
SITE STN # 56	MM (m)	13.64	7.08	3.79	2.47
	MM-RT_1 (cm)	0.43	0.46	0.22	0.15

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/20/70

	STATION	STATION	STATION
#	52	56	58
RELEASE TIME	19:29	19:32	19:38
ALTITUDE (km)	16.0	13.5	16.6
AZIMUTH	69.8	70.3	69.0
ELEVATION	7.8	7.8	8.0
RANGE (km)	118.3	100.2	119.0

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 1/20/70 TIME 19:29

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	130	-2.92	-1.37	-.67	-.41
	140	-3.27	-1.46	-.69	-.41
	150	-3.58	-1.55	-.71	-.41
	160	-3.85	-1.62	-.73	-.41
	170	-4.08	-1.67	-.74	-.42
	180	-4.24	-1.71	-.75	-.42
	190	-4.08	-1.67	-.74	-.42
	200	-3.85	-1.62	-.73	-.41
	210	-3.58	-1.55	-.71	-.41
SITE STN # 52	MM (m)	13.60	7.04	3.77	2.46
	MM-RT_1 (cm)	0.89	0.37	0.14	0.09

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 56 RELEASE DATE 1/20/70 TIME 19:32

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	0	0.02	-.52	-.45	-.37
	10	0.12	-.49	-.44	-.36
	20	0.15	-.49	-.44	-.36
	30	0.12	-.51	-.45	-.36
	40	0.02	-.54	-.46	-.36
	50	-.15	-.59	-.47	-.36
	60	-.37	-.65	-.48	-.37
	70	-.65	-.73	-.50	-.37
	80	-.97	-.82	-.52	-.37
SITE STN # 56	MM (m)	13.62	7.06	3.78	2.47
	MM-RT_1 (cm)	1.25	0.48	0.18	0.11

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 58 RELEASE DATE 1/20/70 TIME 19:38

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	260	-1.83	-1.10	-.61	-.40
	270	-1.45	-.99	-.58	-.40
	280	-1.10	-.89	-.56	-.39
	290	-.77	-.80	-.54	-.39
	300	-.50	-.72	-.52	-.39
	310	-.27	-.66	-.50	-.39
	320	-.10	-.61	-.49	-.39
	330	-.00	-.57	-.48	-.39
SITE STN # 58	MM (m)	13.62	7.06	3.77	2.47
	MM-RT_1 (cm)	0.66	0.24	0.06	0.02

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/15/70

	STATION	STATION	STATION
#	52	54	56
RELEASE TIME	17:32	17:32	17:30
ALTITUDE (km)	15.9	16.0	16.1
AZIMUTH	92.5	93.8	82.2
ELEVATION	10.7	9.7	10.3
RANGE (km)	85.4	95.4	90.1

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DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 1/15/70 TIME 17:32

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	110	-1.15	0.17	0.34	0.33
	120	-1.32	0.13	0.33	0.33
	130	-1.38	0.11	0.33	0.33
	140	-1.35	0.11	0.33	0.33
	150	-1.22	0.15	0.33	0.33
	160	-1.00	0.21	0.35	0.33
	170	-.69	0.29	0.37	0.34
	180	-.31	0.39	0.39	0.33
	190	-.69	0.29	0.37	0.34
SITE STN # 52	MM (m)	13.81	7.16	3.83	2.50
	$MM-RT_1$ (cm)	0.76	0.33	0.11	0.07

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 54 RELEASE DATE 1/15/70 TIME 17:32

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	230	-1.13	0.24	0.39	0.38
	240	-1.07	0.25	0.40	0.38
	250	-.91	0.30	0.41	0.38
	260	-.67	0.37	0.42	0.38
	270	-.34	0.46	0.45	0.39
	280	0.07	0.57	0.47	0.39
	290	0.54	0.70	0.50	0.39
	300	1.05	0.84	0.54	0.40
	310	1.60	1.00	0.57	0.40
SITE STN # 54	MM (m)	13.81	7.16	3.83	2 50
	MM·RT_1 (cm)	0.63	0.32	0.12	0.07

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 56 RELEASE DATE 1/15/70 TIME 17:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	-30	2.25	1.04	0.51	0.31
	-20	2.78	1.19	0.54	0.32
	-10	3.28	1.32	0.58	0.32
	0	3.72	1.45	0.60	0.32
	10	3.28	1.32	0.58	0.32
	20	2.78	1.19	0.54	0.32
	30	2.25	1.04	0.51	0.31
	40	1.69	0.89	0.47	0.31
	50	1.13	0.74	0.44	0.31
SITE STN # 56	MM (m)	13.80	7.15	3.83	2.50
	MM-RT_1 (cm)	0.58	0.27	.09	.05

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/27/70

	STATION	STATION	STATION
#	52	55	58
RELEASE TIME	6:29	6:30	6:35
ALTITUDE (km)	15.8	13.6	16.0
AZIMUTH	93.8	99.9	92.1
ELEVATION	7.2	6.0	8.3
RANGE (km)	126.2	130.5	111.4

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 1/27/70 TIME 6:29

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	20	2.16	0.65	0.16	0.00
	30	1.89	0.57	0.14	-.00
	40	1.56	0.47	0.12	0.01
	50	1.17	0.35	0.09	0.01
	60	0.75	0.22	0.06	0.01
	70	0.31	0.09	0.03	0.01
	80	-.15	-.04	-.01	-.01
	90	-.60	-.18	-.04	-.01
SITE STN # 52	MM (m)	13.55	7.02	3.76	2.46
	MM-RT_1 (cm)	1.25	0.51	0.20	0.13

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 55 RELEASE DATE 1/27/70 TIME 6:30

SITE ALTITUDE (m) 140. WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	130	-2.36	-.77	-.23	-.05
	140	-2.60	-.84	-.24	-.06
	150	-2.78	-.89	-.26	-.05
	160	-2.89	-.92	-.26	-.06
	170	-2.92	-.92	-.26	-.06
	180	-2.87	-.91	-.26	-.06
	190	-2.91	-.92	-.26	-.06
	200	-2.89	-.92	-.26	-.06
SITE STN # 55	MM (m)	13.56	7.03	3.76	2.46
	$MM-RT_1$ (cm)	0.06	0.23	0.10	0.06

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 58 RELEASE DATE 1/27/70 TIME 6:35

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	270	-.85	-.32	-.12	-.03
	280	-.40	-.19	-.09	-.03
	290	0.06	-.05	-.05	-.02
	300	0.50	0.08	-.02	-.02
	310	0.92	0.21	0.01	-.02
	320	1.30	0.32	0.04	-.02
	330	1.63	0.42	0.06	-.01
	340	1.90	0.50	0.08	-.01
SITE STN # 58	MM (m)	13.54	7.02	3.76	2.46
	MM-RT_1 (cm)	0.11	0.19	0.07	0.04

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/20/70

	STATION	STATION	STATION
#	52	53	54
RELEASE TIME	1:30	1:30	1:30
ALTITUDE (km)	16.0	15.9	13.3
AZIMUTH	73.0	71.7	71.3
ELEVATION	6.5	4.4	5.7
RANGE (km)	140.1	206.9	134.0

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 1/20/70 TIME 1:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	60	1.53	0.52	0.16	0.06
	70	0.61	0.26	0.10	0.05
	80	-.31	0.00	0.04	0.05
	90	-1.22	-.25	-.02	0.04
	100	-2.07	-.49	-.08	0.03
	110	-2.86	-.72	-.13	0.03
	120	-3.54	-.91	-.18	0.03
	130	-4.11	-1.08	-.22	0.02
	140	-4.54	-1.20	-.25	0.02
SITE STN # 52	MM (m)	13.71	7.10	3.80	2.48
	MM-RT_1 (cm)	1.21	0.46	0.17	0.11

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 53 RELEASE DATE 1/20/70 TIME 1:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	210	-4.93	-1.36	-.31	-.02
	220	-4.64	-1.27	-.29	-.02
	230	-4.21	-1.15	-.26	-.02
	240	-3.65	-.98	-.22	-.01
	250	-2.97	-.79	-.17	-.01
	260	-2.20	-.57	-.12	-.01
	270	-1.34	-.33	-.06	-.01
	280	-.44	-.07	-.00	-.00
	290	0.48	0.18	0.06	0.01
SITE STN # 53	MM (m)	13.67	7.08	3.79	2.48
	MM-RT_1 (cm)	1.08	0.45	0.17	0.11

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 54 RELEASE DATE 1/20/70 TIME 1:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	-30	3.74	1.08	0.27	0.04
	-20	4.35	1.24	0.30	0.04
	-10	4.83	1.37	0.34	0.04
	0	5.16	1.46	0.36	0.05
	10	4.83	1.37	0.34	0.04
	20	4.35	1.24	0.30	0.04
	30	3.74	1.08	0.27	0.04
	40	3.01	0.88	0.22	0.03
SITE STN # 54	MM (m)	13.68	7.09	3.79	2.48
	MM-RT_1 (cm)	1.28	0.56	0.23	0.14

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/20/70

	STATION	STATION	STATION
#	52	54	55
RELEASE TIME	17:31	17:30	17:36
ALTITUDE (km)	16.0	13.0	17.3
AZIMUTH	69.5	75.3	74.2
ELEVATION	8.6	8.7	8.5
RANGE (km)	106.8	85.9	117.0

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 1/20/70 TIME 17:31

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	130	0.56	0.43	0.29	0.21
	140	0.20	0.33	0.28	0.22
	150	-.11	0.24	0.25	0.20
	160	-.37	0.16	0.23	0.20
	170	-.58	0.10	0.21	0.21
	180	-.72	0.06	0.20	0.21
	190	-.58	0.10	0.21	0.21
	200	-.37	0.17	0.23	0.20
	210	-.11	0.24	0.25	0.20
SITE STN # 52	MM (m)	13.54	7.02	3.75	2.45
	MM-RT_1 (cm)	0.80	0.34	0.12	0.07

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 54 RELEASE DATE 1/20/70 TIME 17:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	260	-.44	-.28	-.17	-.14
	270	-.04	-.17	-.15	-.14
	280	0.34	-.06	-.12	-.14
	290	0.69	0.04	-.10	-.14
	300	1.00	0.12	-.08	-.13
	310	1.26	0.19	-.06	-.13
	320	1.46	0.24	-.05	-.13
	330	1.60	0.28	-.04	-.13
SITE STN # 54	MM (m)	13.54	7.02	3.75	2.45
	MM-RT_1 (cm)	1.32	0.57	0.24	0.15

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 55 RELEASE DATE 1/20/70 TIME 17:36

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	10	1.53	0.22	-.08	-.14
	20	1.53	0.22	-.08	-.14
	30	1.47	0.20	-.08	-.14
	40	1.33	0.17	-.09	-.14
	50	1.13	0.12	-.10	-.15
	60	0.87	0.05	-.12	-.15
	70	0.56	-.04	-.14	-.15
	80	0.21	-.14	-.16	-.15
SITE STN # 55	MM (m)	13.54	7.02	3.75	2.45
	MM-RT_1 (cm)	0.57	0.23	0.06	0.04

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/19/70

	STATION	STATION	STATION
#	53	55	57
RELEASE TIME	21.35	21.32	21.30
ALTITUDE (km)	16.0	16.0	14.9
AZIMUTH	71.0	72.5	67.8
ELEVATION	6.2	6.1	7.3
RANGE (km)	147.5	150.7	117.3

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 53 RELEASE DATE 1/19/70 TIME 21:35

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	170	-6.04	-2.38	-1.02	-.56
	180	-6.27	-2.45	-1.04	-.56
	190	-6.04	-2.38	-1.02	-.56
	200	-5.72	-2.29	-1.00	-.56
	210	-5.31	-2.18	-.98	-.56
	220	-4.82	-2.05	-.94	-.55
	230	-4.28	-1.90	-.91	-.55
	240	-3.69	-1.74	-.87	-.55
	250	-3.08	-1.58	-.83	-.54
SITE STN # 53	MM (m)	13.54	7.02	3.75	2.45
	MM-RT_1 (cm)	0.84	0.36	0.13	0.10

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 55 RELEASE DATE 1/19/70 TIME 21:32

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10°	20°	40°	80°
RT_1-RT_3 (cm)	40	0.50	-.59	-.59	-.50
	50	0.20	-.67	-.61	-.50
	60	-.19	-.77	-.63	-.50
	70	-.66	-.90	-.66	-.51
	80	-1.19	-1.04	-.69	-.51
	90	-1.77	-1.20	-.73	-.51
	100	-2.38	-1.37	-.77	-.52
	110	-3.00	-1.53	-.81	-.52
	120	-3.61	-1.70	-.85	-.52
SITE STN # 55	MM (m)	[illegible].56	7.03	3.76	2.46
	MM-RT_1 (cm)	0.74	0.32	0.11	0.08

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 57 RELEASE DATE 1/19/70 TIME 21:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	-30	0.81	-.49	-.55	-.50
	-20	0.91	-.46	-.54	-.51
	-10	0.89	-.47	-.55	-.51
	0	0.77	-.50	-.55	-.50
	10	0.89	-.47	-.55	-.51
	20	0.91	-.46	-.54	-.50
	30	0.81	-.49	-.55	-.50
	40	0.60	-.54	-.56	-.50
SITE STN # 57	MM (m)	13.57	7.03	3.76	2.46
	MM-RT_1 (cm)	1.07	0.28	0.06	0.04

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/19/70

	STATION	STATION	STATION
#	52	55	58
RELEASE TIME	17:31	17:32	17:30
ALTITUDE (km)	15.8	16.1	16.0
AZIMUTH	70.5	72.4	71.5
ELEVATION	6.0	6.3	5.7
RANGE (km)	152.1	146.5	160.5

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 1/19/70 TIME 17:31

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	120	-1.40	-.94	-.57	-.42
	130	-1.53	-.97	-.57	-.41
	140	-1.68	-1.01	-.58	-.42
	150	-1.85	-1.05	-.59	-.41
	160	-2.02	-1.09	-.60	-.43
	170	-2.20	-1.14	-.61	-.42
	180	-2.38	-1.19	-.63	-.43
	190	-2.20	-1.14	-.61	-.42
	200	-2.02	-1.09	-.60	-.43
SITE STN # 52	MM (m)	13.60	7.04	3.77	2.46
	MM-RT_1 (cm)	1.16	0.39	0.13	0.08

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 55 RELEASE DATE 1/19/70 TIME 17:32

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10°	20°	40°	80°
RT_1-RT_3 (cm)	10	-1.71	-1.02	-.57	-.38
	20	-1.55	-.98	-.56	-.38
	30	-1.40	-.94	-.55	-.37
	40	-1.28	-.90	-.55	-.37
	50	-1.18	-.88	-.54	-.37
	60	-1.10	-.85	-.53	-.37
	70	-1.06	-.84	-.53	-.36
	80	-1.05	-.83	-.53	-.36
	90	-1.07	-.83	-.53	-.35
SITE STN # 55	MM (m)	13.58	7.04	3.76	2.46
	MM-RT_1 (cm)	0.72	0.31	0.10	0.07

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 58 RELEASE DATE 1/19/70 TIME 17:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10°	20°	40°	80°
RT_1-RT_3 (cm)	270	-1.35	-.99	-.61	-.43
	280	-1.33	-.98	-.61	-.45
	290	-1.34	-.99	-.61	-.44
	300	-1.38	-1.00	-.62	-.43
	310	-1.45	-1.02	-.62	-.44
	320	-1.54	-1.05	-.63	-.44
	330	-1.67	-1.08	-.64	-.43
	340	-1.81	-1.12	-.64	-.43
	350	-1.97	-1.17	-.66	-.44
SITE STN # 58	MM (m)	13.56	7.03	3.76	2.46
	MM $\cdot RT_1$ (cm)	0.87	0.20	0.16	0.01

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 2/13/70

	STATION	STATION	STATION
#	52	53	56
RELEASE TIME	17:30	17:30	17:30
ALTITUDE (km)	15.8	15.0	13.1
AZIMUTH	89.7	85.6	82.8
ELEVATION	6.6	4.6	5.9
RANGE (km)	137.5	187.0	127.4

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 2/13/70 TIME 17:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	90	0.40	0.35	0.23	0.16
	100	- .09	0.22	0.19	0.16
	110	- .55	0.09	0.17	0.15
	120	- .97	-.02	0.14	0.15
	130	-1.32	-.11	0.12	0.14
	140	-1.61	-.19	0.10	0.15
	150	-1.82	-.25	0.08	0.15
	160	-1.95	-.29	0.07	0.14
	170	-2.00	-.30	0.07	0.14
SITE STN # 52	MM (m)	13.67	7.09	3.79	2.48
	MM-RT_1 (cm)	0.49	0.35	0.14	0.08

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 53 RELEASE DATE 2/13/70 TIME 17:30

SITE ALTITUDE (m) 35 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	200	-3.53	-1.13	-.38	-.17
	210	-3.34	-1.08	-.37	-.16
	220	-3.08	-1.01	-.35	-.16
	230	-2.74	-.92	-.33	-.16
	240	-2.35	-.81	-.30	-.16
	250	-1.90	-.69	-.28	-.16
	260	-1.42	-.56	-.24	-.15
	270	- .91	-.43	-.21	-.15
	280	-.40	-.29	-.18	-.14
SITE STN # 53	MM (m)	13.64	7.07	3.78	2.47
	MM-RT_1 (cm)	0.88	0.53	0.24	0.17

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 56 RELEASE DATE 2/13/70 TIME 17:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	340	2.88	0.86	0.26	0.09
	350	3.09	0.92	0.27	0.09
	0	3.22	0.95	0.28	0.08
	10	3.09	0.92	0.27	0.09
	20	2.88	0.86	0.26	0.09
	30	2.59	0.79	0.24	0.08
	40	2.23	0.69	0.22	0.08
	50	1.81	0.58	0.19	0.07
	60	1.35	0.46	0.17	0.07
SITE STN # 56	MM (m)	13.64	7.08	3.79	2.47
	MM·RT_1 (cm)	0.42	0.45	0.22	0.15

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/21/70

	STATION	STATION	STATION
#	52	53	56
RELEASE TIME	13:30	13:35	13:30
ALTITUDE (km)	12.9	15.8	16.0
AZIMUTH	102.3	101.2	103.5
ELEVATION	4.2	5.2	6.3
RANGE (km)	177.1	172.8	144.8

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 1/21/70 TIME 13:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	90	1.74	2.08	1.47	1.10
	100	1.40	1.98	1.45	1.10
	110	1.21	1.93	1.43	1.10
	120	1.16	1.91	1.43	1.10
	130	1.26	1.94	1.44	1.10
	140	1.50	2.00	1.45	1.10
	150	1.89	2.10	1.47	1.10
	160	2.40	2.23	1.51	1.11
SITE STN # 52	MM (m)	13.74	7.12	3.81	2.49
	MM-RT_1 (cm)	1.31	0.54	0.21	0.14

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 53 RELEASE DATE 1/21/70 TIME 13:35

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10°	20°	40°	80°
RT_1-RT_3 (cm)	210	3.19	2.77	1.83	1.34
	220	2.79	2.67	1.81	1.34
	230	2.54	2.60	1.79	1.34
	240	2.42	2.57	1.79	1.34
	250	2.46	2.59	1.79	1.34
	260	2.65	2.64	1.80	1.34
	270	2.97	2.73	1.82	1.34
	280	3.43	2.86	1.85	1.34
SITE STN # 53	MM (m)	13.66	7.07	3.78	2.47
	MM-RT_1 (cm)	1.00	0.49	0.20	0.14

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 56 RELEASE DATE 1/21/70 TIME 13:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	-20	8.09	4.16	2.19	1.41
	-10	8.91	4.38	2.24	1.42
	0	9.69	4.59	2.29	1.43
	10	8.91	4.38	2.24	1.42
	20	8.09	4.16	2.19	1.41
	30	7.24	3.93	2.13	1.41
	40	6.40	3.71	2.08	1.40
	50	5.59	3.49	2.02	1.40
SITE STN # 56	MM (m)	13.75	7.12	3.81	2.49
	MM-RT_1 (cm)	1.81	0.70	0.29	0.19

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 2/13/70

	STATION	STATION	STATION
#	52	54	56
RELEASE TIME	21:30	21:30	21:30
ALTITUDE (km)	16.0	13.3	13.2
AZIMUTH	89.8	86.6	85.8
ELEVATION	7.2	7.8	7.3
RANGE (km)	127.7	97.6	103.2

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 2/13/70 TIME 21:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	100	-.55	-.02	0.07	0.06
	110	-.95	-.13	0.04	0.06
	120	-1.31	-.23	0.02	0.05
	130	-1.62	-.32	-.00	0.05
	140	-1.86	-.38	-.02	0.05
	150	-2.03	-.43	-.03	0.05
	160	-2.13	-.46	-.04	0.05
	170	-2.15	-.47	-.04	0.04
SITE STN # 52	MM (m)	13.68	7.09	3.79	2.48
	MM-RT_1 (cm)	0.55	0.35	0.14	0.11

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 54 RELEASE DATE 2/13/70 TIME 21:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	230	-1.86	-.45	-.07	0.03
	240	-1.55	-.36	-.05	0.03
	250	-1.20	-.27	-.03	0.03
	260	-.80	-.16	-.00	0.03
	270	-.36	-.04	0.02	0.04
	280	0.09	0.09	0.05	0.04
	290	0.54	0.21	0.08	0.04
	300	0.99	0.33	0.11	0.05
SITE STN # 54	MM (m)	13.65	7.08	3.79	2.47
	MM-RT_1 (cm)	0.68	0.48	0.22	0.15

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 56 RELEASE DATE 2/13/70 TIME 21:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	-20	2.38	0.70	0.20	0.06
	-10	2.59	0.76	0.21	0.06
	0	2.72	0.79	0.22	0.06
	10	2.59	0.76	0.21	0.06
	20	2.38	0.70	0.20	0.06
	30	2.10	0.63	0.18	0.05
	40	1.77	0.54	0.16	0.05
	50	1.38	0.43	0.13	0.05
SITE STN # 56	MM (m)	13.64	7.08	3.79	2.48
	MM-RT_1 (cm)	0.14	0.34	0.16	0.10

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/27/70

	STATION	STATION	STATION
#	52	55	58
RELEASE TIME	4:26	4:30	4:30
ALTITUDE (km)	16.0	16.0	13.2
AZIMUTH	92.1	98.1	89.0
ELEVATION	8.8	7.2	8.2
RANGE (km)	105.1	127.6	93.0

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 1/27/70 TIME 4:26

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	130	-.41	-.01	0.06	0.04
	140	-.60	-.07	0.05	0.04
	150	-.75	-.12	0.03	0.04
	160	-.87	-.16	0.02	0.04
	170	-.95	-.19	0.02	0.04
	180	-.99	-.20	0.01	0.03
	190	-.95	-.19	0.02	0.04
	200	-.87	-.16	0.02	0.04
SITE STN # 52	MM (m)	13.54	7.02	3.76	2.46
	MM·RT_1 (cm)	-.18	0.10	0.03	0.04

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 55 RELEASE DATE 1/27/70 TIME 4:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	20	2.17	0.83	0.32	0.14
	30	2.08	0.81	0.31	0.14
	40	1.95	0.77	0.31	0.14
	50	1.79	0.73	0.30	0.14
	60	1.59	0.67	0.28	0.14
	70	1.37	0.61	0.27	0.14
	80	1.13	0.54	0.25	0.13
	90	0.89	0.47	0.23	0.14
SITE STN # 55	MM (m)	13.53	7.02	3.75	2.45
	MM-RT_1 (cm)	0.21	0.22	0.08	0.07

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 58 RELEASE DATE 1/27/70 TIME 4:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	270	0.57	0.30	0.14	0.09
	280	0.81	0.37	0.16	0.10
	290	1.05	0.44	0.18	0.10
	300	1.27	0.50	0.19	0.10
	310	1.46	0.55	0.20	0.10
	320	1.63	0.60	0.21	0.11
	330	1.76	0.63	0.22	0.10
	340	1.85	0.66	0.23	0.11
SITE STN # 58	MM (m)	13.52	7.01	3.75	2.45
	MM-RT_1 (cm)	0.13	0.21	0.08	0.04

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/26/70

	STATION	STATION	STATION
#	52	53	56
RELEASE TIME	9:30	9:30	9:30
ALTITUDE (km)	16.2	16.1	13.6
AZIMUTH	76.5	77.5	71.0
ELEVATION	6.4	4.9	6.1
RANGE (km)	145.4	187.9	127.7

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 52 RELEASE DATE 1/26/70 TIME 9:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	ELEVATION / AZIMUTH	10°	20°	40°	80°
RT_1-RT_3 (cm)	90	-.96	-.70	-.44	-.30
	100	-1.58	-.87	-.48	-.31
	110	-2.20	-1.04	-.52	-.31
	120	-2.80	-1.21	-.56	-.32
	130	-3.37	-1.37	-.60	-.32
	140	-3.89	-1.51	-.63	-.33
	150	-4.34	-1.64	-.67	-.33
	160	-4.71	-1.72	-.69	-.33
SITE STN # 52	MM (m)	13.44	6.97	3.73	2.44
	MM-RT_1 (cm)	0.19	0.04	-.04	-.03

DATA SUMMARY

Sheet 3: Ray Trace Parameters

SITE STN 53 RELEASE DATE 1/26/70 TIME 9:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10°	20°	40°	80°
RT_1-RT_3 (cm)	210	-4.57	-1.77	-.73	-.37
	220	-4.12	-1.64	-.70	-.37
	230	-3.60	-1.49	-.67	-.36
	240	-3.03	-1.34	-.63	-.36
	250	-2.43	-1.17	-.59	-.36
	260	-1.81	-.99	-.55	-.35
	270	-1.18	-.82	-.51	-.35
	280	-.58	-.66	-.47	-.34
SITE STN # 53	MM (m)	13.42	6.96	3.73	2.44
	$MM-RT_1$ (cm)	-.11	-.02	-.06	-.04

DATA SUMMARY

Sheet 4: Ray Trace Parameters

SITE STN 56 RELEASE DATE 1/26/70 TIME 9:30

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 530

	AZIMUTH \ ELEVATION	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	340	1.79	0.03	-.28	-.32
	350	1.83	0.04	-.28	-.31
	360	1.76	0.02	-.28	-.32
	370	1.83	0.04	-.28	-.31
	380	1.79	0.03	-.28	-.32
	390	1.64	-.01	-.29	-.32
	400	1.39	-.08	-.31	-.32
	410	1.04	-.17	-.33	-.32
SITE STN # 56	MM (m)	13.47	6.99	3.74	2.45
	[illegible]'-RT_1 (cm)	0.0[illegible]	0.05	-.02	-.02

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/19/70

	STATION	STATION	STATION
#	52	55	58
RELEASE TIME	17:31	17:32	17:30
ALTITUDE (km)	15.8	16.1	16.0
AZIMUTH	70.5	72.4	71.5
ELEVATION	6.0	6.3	5.7
RANGE (km)	152.1	146.5	160.5

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 58 RELEASE DATE 1/19/70 TIME 17:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 353

	ELEVATION / AZIMUTH	10°	20°	40°	80°
RT_1-RT_3 (cm)	270	-1.47	-1.07	-.66	-.48
	280	-1.44	-1.06	-.66	-.48
	290	-1.45	-1.07	-.66	-.48
	300	-1.49	-1.08	-.67	-.49
	310	-1.56	[illegible]	-.67	-.48
	320	-1.67	[illegible]	-.68	-.48
	330	-1.80	-1.[illegible]7	-.69	-.48
	340	-1.96	-1.22	-.70	-.48
	350	-2.13	-1.26	-.71	-.48
SITE STN # 58	MM (m)	14.67	7.61	4.07	2.66
	$MM-RT_1$ (cm)	0.80	0.10	-.03	-.01

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/21/70

	STATION	STATION	STATION
#	52	55	56
RELEASE TIME	21:32	21:47	21:34
ALTITUDE (km)	15.9	13.2	12.6
AZIMUTH	106.1	104.3	96.9
ELEVATION	7.2	6.4	5.1
RANGE (km)	127.1	119.1	142.1

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 56 RELEASE DATE 1/21/70 TIME 21:34

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 353

	ELEVATION / AZIMUTH	10^{o}	20^{o}	40^{o}	80^{o}
RT_1-RT_3 (cm)	340	3.35	1.05	0.31	0.09
	350	3.43	1.06	0.32	0.09
	360	3.41	1.05	0.31	0.09
	370	3.43	1.06	0.32	0.09
	380	3.35	1.05	0.31	0.09
	390	3.19	1.01	0.31	0.09
	400	2.94	0.95	0.29	0.08
	410	2.62	0.86	0.27	0.08
SITE STN # 56	MM (m)	14.88	7.71	4.12	2.69
	MM-RT_1 (cm)	1.31	0.68	0.32	0.23

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/21/70

	STATION	STATION	STATION
#	52	54	56
RELEASE TIME	23:36	23:35	23:31
ALTITUDE (km)	15.9	12.9	13.2
AZIMUTH	107.4	105.4	99.3
ELEVATION	7.7	4.9	6.3
RANGE (km)	118.4	149.4	120.4

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 56 RELEASE DATE 1/21/70 TIME 23:31

SITE ALTITUDE (m) 85 WAVE LENGTH (nm) 353

	ELEVATION / AZIMUTH	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	340	5.35	1.79	0.61	0.24
	350	5.29	1.76	0.60	0.24
	360	5.10	1.68	0.58	0.24
	370	5.29	1.76	0.60	0.23
	380	5.35	1.79	0.61	0.24
	390	5.30	1.79	0.61	0.23
	400	5.12	1.76	0.60	0.23
	410	4.83	1.69	0.59	0.23
SITE STN # 56	MM (m)	14.91	7.73	4.13	2.70
	MM-RT_1 (cm)	1.46	0.59	0.25	0.17

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/27/70

	STATION	STATION	STATION
#	52	55	58
RELEASE TIME	2:30	2:30	2:30
ALTITUDE (km)	15.4	15.9	16.0
AZIMUTH	86.6	93.5	84.6
ELEVATION	7.8	7.0	10.3
RANGE (km)	113.6	130.6	90.0

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 58 RELEASE DATE 1/27/70 TIME 2:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 353

	AZIMUTH \ ELEVATION	10°	20°	40°	80°
RT_1-RT_3 (cm)	270	-.07	-.16	-.16	-.14
	280	0.07	-.15	-.15	-.14
	290	0.19	-.14	-.14	-.13
	300	0.28	-.14	-.14	-.13
	310	0.33	-.14	-.14	-.13
	320	0.36	-.13	-.13	-.13
	330	0.36	-.14	-.14	-.13
	340	0.32	-.14	-.14	-.13
SITE STN # 58	MM (m)	14.62	7.58	4.06	2.65
	MM-RT_1 (cm)	-.17	0.06	0.01	0.03

DATA SUMMARY

Sheet 1: Radiosonde Parameters

Release Date 1/30/70

	STATION	STATION	STATION
#	52	55	56
RELEASE TIME	21:30	21:30	21:30
ALTITUDE (km)	15.9	16.1	13.1
AZIMUTH	76.7	82.4	73.0
ELEVATION	7.5	7.1	6.5
RANGE (km)	121.4	129.4	118.4

DATA SUMMARY

Sheet 2: Ray Trace Parameters

SITE STN 56 RELEASE DATE 1/30/70 TIME 21:30

SITE ALTITUDE (m) 140 WAVE LENGTH (nm) 353

	AZIMUTH \ ELEVATION	10^o	20^o	40^o	80^o
RT_1-RT_3 (cm)	300	0.97	-.97	-1.05	-.93
	310	0.97	-.98	-1.05	-.92
	320	0.79	-1.03	-1.06	-.93
	330	0.43	-1.14	-1.09	-.93
	340	-.09	-1.29	-1.12	-.93
	350	-.75	-1.49	-1.17	-.94
	360	-1.54	-1.72	-1.23	-.94
	370	-.75	-1.49	-1.17	-.94
SITE STN # 56	MM (m)	14.67	7.60	4.07	2.66
	MM-RT_1 (cm)	0.75	0.52	0.25	0.18

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